Title: Algebrability of classes of Sierpiński-Zygmund-like functions
Author: Artur Bartoszewicz

We say that a subset $E$ of a commutative linear algebra $B$ is strongly $\kappa$-algebrable if there exists a $\kappa$-generated free algebra $A$ contained in $E \cup \{0\}$.

**Theorem 1** (Sierpiński-Zygmund) There exists a function $f : \mathbb{R} \to \mathbb{R}$ such that, for any set $Z \subset \mathbb{R}$ of cardinality continuum, the restriction $f|_Z$ is not a Borel map (and, in particular, not continuous).

**Theorem 2** [1] The set of Sierpiński-Zygmund functions is strongly $\kappa$-algebrable, provided there exists a family of $\kappa$ almost disjoint subsets of $c$.

We say that a function $f : \mathbb{R} \to \mathbb{R}$ is a strong Sierpiński-Zygmund function, if for every set $A \subseteq \mathbb{R}$ of cardinality $\omega_1$ the restriction $f|_A$ is not a Borel map. Let us denote by $sSZ(\mathbb{R})$ the set of all strong Sierpiński-Zygmund functions.

**Theorem 3** [2] If $sSZ(\mathbb{R}) \neq \emptyset$, then it is strongly $c$-algebrable.

One can ask, if the assumption of the above Theorem can be fulfilled under the negation of the Continuum Hypothesis. Gruenhage proved that if model $V$ of ZFC satisfies $2^\omega = \kappa$ and $\lambda \geq \kappa$ is a cardinal with $\lambda^\omega = \lambda$, then, after adding $\lambda$ Cohen or random reals to $V$, there exists $sSZ$ function in the extension.

By $SZ(\Phi) \subseteq \mathbb{k}^\mathbb{K}$ we denote the family of all functions $f : \mathbb{K} \to \mathbb{K}$ with $f|_Z \notin \Phi$ for any $Z \in [\mathbb{K}]^c$ (the symbol $[\mathbb{K}]^c$ stands for the family of all subsets of $\mathbb{K}$ that have cardinality $c$).

**Theorem 4** [3] Assume CH. The family $ES(\mathbb{C}) \cap (SZ(\mathbb{C}) \setminus SZ(Bor)) \subseteq \mathbb{C}^c$ is strongly $c$-algebrable.

**References**

