Title: Algebrability of classes of Sierpiński-Zygmund-like functions
Author: Artur Bartoszewicz

We say that a subset E of a commutative linear algebra B is strongly κ-algebrable if there exists a κ-generated free algebra A contained in $E \cup \{0\}$.

Theorem 1 *(Sierpiński-Zygmund) There exists a function $f\colon \mathbb{R} \to \mathbb{R}$ such that, for any set $Z \subset \mathbb{R}$ of cardinality continuum, the restriction $f|_Z$ is not a Borel map (and, in particular, not continuous).*

Theorem 2 *[1] The set of Sierpiński-Zygmund functions is strongly κ-algebrable, provided there exists a family of κ almost disjoint subsets of $\mathfrak{c}$.*

We say that a function $f : \mathbb{R} \to \mathbb{R}$ is a strong Sierpiński-Zygmund function, if for every set $A \subseteq \mathbb{R}$ of cardinality ω_1 the restriction $f|_A$ is not a Borel map. Let us denote by $s\mathcal{SZ}(\mathbb{R})$ the set of all strong Sierpiński-Zygmund functions.

Theorem 3 *[2] If $s\mathcal{SZ}(\mathbb{R}) \neq \emptyset$, then it is strongly $\mathfrak{c}$-algebrable.*

One can ask, if the assumption of the above Theorem can be fulfilled under the negation of the Continuum Hypothesis. Gruenhage proved that if model V of ZFC satisfies $2^\omega = \kappa$ and $\lambda \geq \kappa$ is a cardinal with $\lambda^\omega = \lambda$, then, after adding λ Cohen or random reals to V, there exists $s\mathcal{SZ}$ function in the extension.

By $\mathcal{SZ}(\Phi) \subseteq \mathbb{K}^{\mathbb{K}}$ we denote the family of all functions $f : \mathbb{K} \to \mathbb{K}$ with $f|Z \notin \Phi$ for any $Z \in [\mathbb{K}]^{\mathfrak{c}}$ (the symbol $[\mathbb{K}]^{\mathfrak{c}}$ stands for the family of all subsets of $\mathbb{K}$ that have cardinality $\mathfrak{c}$).

Theorem 4 *[3] Assume CH. The family $\mathcal{ES}(\mathbb{C}) \cap (\mathcal{SZ}(\mathcal{C}) \setminus \mathcal{SZ}(\mathcal{B}or)) \subseteq \mathbb{C}^{\mathbb{C}}$ is strongly $\mathfrak{c}$-algebrable.*

References

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