

Some Connections among Topology, Model Theory, and Set Theory

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Both the Baire Category Theorem (BCT) and the Omitting Types Theorem (OTT) of Model Theory are used when one wants to satisfy countably many requirements simultaneously, often in situations when compactness in either the topological or model-theoretic sense is not available. It has been known at least since the work of Morley in the 1970's that there is a strong connection between the BCT and the OTT, but the details have been unclear. We clarify that connection here, noting that what is really involved is spaces being completely Baire, i.e. having all closed subspaces Baire. Building on the work of R. Knight, we associate to each topological space an abstract logic. This enables us to transfer pathological examples from topology to model theory. In particular, we tease out different forms of the OTT which are equivalent for first-order logic, but not for more general logics. In particular, there is a game version of the OTT analogous to weak α -favourability which is not equivalent to the OTT for a certain logic constructed from a non-meagre P -filter. It is not known whether such filters exist in ZFC, but they do unless there is a large cardinal in an inner model. By requiring logics to be definable in a Descriptive Set Theory sense and assuming corresponding Determinacy axioms, we can use results of Medini and Zdomskyy to show that the game version of the OTT is equivalent to the usual version for such logics.