

# GENERAL APPROACH TO MICROSCOPIC-TYPE SETS

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A new concept of small subsets of the real line is presented. It is a generalization of different kinds of microscopic sets considered previously. We compare these sets with Lebesgue null sets, with sets of strong measure zero. Contrary to classical version we are able to get here families of fractals - sets with fractional Hausdorff dimension.

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