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Boolean algebras with the approximation property and twisted sums of Banach spaces

ABSTRACT. This is a joint research with **Witold Marciszewski** (Warsaw).

We say that a Boolean algebra \mathfrak{A} has the approximation property (AP) if every sequence of “nearly additive” real-valued functions on \mathfrak{A} can be suitably approximated by a bounded sequence of signed measures. We have proved that

Theorem 1. *If K is compact zerodimensional and the algebra $\text{Clop}(K)$ has AP then every twisted sum of c_0 and $C(K)$ is trivial, i.e. whenever c_0 is embedded into a Banach space X so that X/c_0 is isomorphic to $C(K)$ then c_0 is complemented in X .*

Theorem 2. *Assuming $MA(\omega_1)$, the algebra $\mathfrak{A} = \text{Clop}(2^{\omega_1})$ has AP and, consequently, c_0 admits no nontrivial twisted sum with $C(2^{\omega_1})$.*

This gives a (consistent) negative answer to a problem posed by Cabello Sánchez, Castillo, Kalton, and Yost. There is another uncountable \mathfrak{A} with AP such that its Stone space is scattered. It is unclear, however, how large the class of algebras with AP can be.