

Connected and/or topological group pd-examples

The pinning down number $pd(X)$ of a topological space X is the smallest cardinal κ such that for every neighborhood assignment \mathcal{U} on X there is a set of size κ that meets every member of \mathcal{U} . Clearly, $pd(X) \leq d(X)$ and we call X a *pd-example* if $pd(X) < d(X)$. We denote by \mathbf{S} the class of all singular cardinals that are not strong limit. It was proved by Juhász–Soukup–Szentmiklóssy in 2015 that TFAE:

- (1) $\mathbf{S} \neq \emptyset$;
- (2) there is a 0-dimensional T_2 pd-example;
- (3) there is a T_2 pd-example.

The same triple, together with Jan van Mill, has produced recently pd-examples with further interesting topological properties:

We showed that $\mathbf{S} \neq \emptyset$ is also equivalent to the existence of a connected and locally connected T_3 pd-example, as well as to the existence of an abelian T_2 topological group pd-example.

However, $\mathbf{S} \neq \emptyset$ in itself is not sufficient to imply the existence of a connected $T_{3.5}$ pd-example. But if there is $\mu \in \mathbf{S}$ with $\mu \geq \mathfrak{c}$ then there is an abelian T_2 topological group (hence $T_{3.5}$) pd-example which is also arcwise connected and locally arcwise connected. Finally, the same assumption $\mathbf{S} \setminus \mathfrak{c} \neq \emptyset$ even implies that there is a locally convex topological vector space pd-example.

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