Title: On Steinhaus and Smital properties for measures and sets
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Assume that $\langle X, +\rangle$ is a locally compact Polish abelian group. We say that a Borel measure μ on X has *the Steinhaus property* if the interior of the set $A - A$ is nonempty for any Borel set A of positive measure. A measure μ has *the Smital property* if for any Borel set A of positive measure and any dense set D, the set $A + D$ has a full measure.

Theorem 1 *For any σ-finite Borel measure μ on X, the following conditions are equivalent:*

1. *μ is absolutely continuous to the Haar measure λ on X,*
2. *μ has the Steinhaus property,*
3. *μ has the Smital property.*

We say that a set $A \subset X$ has *the Steinhaus property* if the interior of the set $A - A$ is nonempty. A set $A \subset X$ has *the Smital property* if for any dense set D, $\lambda (X \setminus (A + D)) = 0$. We will discuss relationships between those notions. In particular, we will show that if X is separable then any set having the Smital property has the Steinhaus property. However, if X is σ-compact and is not discrete then there are sets with the Steinhaus property and without the Smital property. We will also construct a set A with $\lambda_* (A) = 0$ such that for any dense set D, the set $X \setminus (A + D)$ has at most one point.