Set Theoretic Methods in Topology and Analysis
03.09.2017 - 09.09.2017, Będlewo

Rainwater sets and $C^b(X)$ weakly K-analytic

M. López-Pellicer[(1)], joint work with J.C. Ferrando[(2)] and J. Kąkol[(3)]

A subset Y of the dual closed unit ball B_{E^*} of a Banach space E is a *Rainwater set* for E if every bounded sequence of E that converges pointwise on Y converges weakly in E ([2]). Each James boundary in B_{E^*} is a Rainwater set ([3]).

Following [1], we present some topological properties of Rainwater sets for the Banach space $C^b(X)$ of the real continuous bounded functions defined on a completely regular topological space X, endowed with the supremum norm. These results enable to characterize that $C^b(X)$ is weakly K-analytic (resp. weakly countably determined, WCD for short) if and only if there exists a Rainwater set Y for $C^b(X)$ such that $(C^b(X), \sigma_Y)$ is both K-analytic (resp. a Lindelöf Σ-space) and angelic, where σ_Y denotes the topology on $C^b(X)$ of the pointwise convergence on Y.

As applications we get: 1) If X is pseudocompact then $C_p(X)$ is K-analytic (resp. a Lindelöf Σ-space) if and only if the Banach space $C(X)$ is weakly K-analytic (resp. WCD). 2) If X is compact and Y is a G_δ-dense subspace, then X is Talagrand compact (resp. Gul'ko compact) if and only if $(C(X), \sigma_Y)$ is K-analytic (resp. Lindelöf Σ). Notice that for $Y := X$ we get classic Talagrand's theorem. For examples, other results and applications see [1].

References

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(1) Department of Applied Mathematics and IUMPA, Universitat Politècnica de València, Spain, mlopezpe@mat.upv.es

(2) Centro de Investigación Operativa, Universidad Miguel Henández de Elche, Spain, jc.ferrando@umh.es

(3) Faculty of Mathematics and Informatics, A. Mickiewicz University Poznań, Poland, kakol@amu.edu.pl