# TOPOLOGICAL GROUPS WHOSE CLOSED SUBGROUPS ARE SEPARABLE

MIKHAIL TKACHENKO
*UNIVERSIDAD AUTÓNOMA METROPOLITANA, MEXICO CITY*

*MSC (2010):* 22A05, 54H11, 54A25

By a theorem of Comfort and Itzkowitz (1977), all closed subgroups of a separable locally compact topological group are separable. Another useful fact established by Comfort (1984) states that a compact topological group is separable if and only if its weight is less than or equal to $2^\omega$. The latter fact, however, cannot be extended to locally compact groups.

It turns out that Comfort's observation on compact groups remains valid for groups from the much wider class of *almost connected pro-Lie* groups which contains all connected locally compact groups and their products. In particular, every closed subgroup of a separable almost connected pro-Lie group is also separable (Leiderman–Morris–Tkachenko, 2016).

These results make it natural to consider the class $\mathcal{CSS}$ of topological groups in which all closed subgroups are separable. Clearly this class of groups is closed under taking continuous homomorphic images and closed subgroups. Quite surprisingly, this class is not finitely productive: There exist pseudocompact topological abelian groups $G$ and $H$ in $\mathcal{CSS}$ such that the product group $G \times H$ contains a closed non-separable subgroup (Leiderman–Tkachenko, 2017). The construction of the groups $G$ and $H$ requires the use of the hypothesis $2^{\omega_1} = 2^\omega$.

In the lecture we will present more information on the class $\mathcal{CSS}$ and give a solution (in ZFC) to the problem on the finite productivity of this class. Several open problem will be formulated as well.