DIMENSION DROP PHENOMENA AND COMPACT SUPPORTS IN NONCOMMUTATIVE TOPOLOGY

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Abstract: If X is locally compact Hausdorff space, a function $f \in C_0(X)$ has compact support, if and only if there is a norm one $g \in C_0(X)$ such that $f = fg$, which is abbreviated $f \ll g$ (f is *way below* g). Functions with compact support ordered by $\ll$ form a net $(e_\lambda)_{\lambda \in \Lambda}$ such that $\|f - fe_\lambda\| \to 0$ for $\lambda \in \Lambda$ and all $f \in C_0(X)$.

We address the question of the existence of such a net (called an almost idempotent approximate identity in Blackadar's textbook) for noncommutative C*-algebras. It is well-know that they exist in all separable C*-algebras. In the positive direction we prove that they also exist if the density of the algebra is ω_1.

We produce first examples of C*-algebras without such almost idempotent approximate identity (of density 2^κ where $\kappa = \min\{\lambda : 2^\lambda > 2^\omega\}$). It is a subalgebra of D^T where T is an appropriate tree and D is a subalgebra of continuous functions on $\{1/n : n \in \mathbb{N}\} \cup \{0\}$ into 2×2-matrices with coordinatewise operations, where the dimension drop phenomenon may occur. Such algebras can be represented on $\mathcal{B}(\ell_2(2^\omega))$ (the noncommutative version of $\wp(2^\omega)$). If there is a Canadian tree (a tree with levels $\leq \omega_1$ and height ω_1 and with more than ω_1 uncountable branches) they can be represented on $\mathcal{B}(\ell_2(\omega_1))$, and if there is a Canadian tree and an uncountable Q-set, they can be represented as an algebra of operators on the separable Hilbert space $\ell_2(\mathbb{N})$.

These are also first examples of scattered C*-algebras (corresponding to scattered locally compact spaces) without directed family of finite-dimensional subalgebras whose union is dense.

No knowledge beyond multiplication of 2×2-matrices of noncommutative mathematics is needed to follow the talk, as all noncommutative C*-algebras we consider in the talk are subalgebras of continuous functions on locally compact spaces into 2×2-matrices with coordinatewise operations.

These are results of a joint research project with Tristan Bice, the joint preprint should appear at matharxiv by the time of the conference.

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