A QUANTITATIVE GENERALIZATION OF PRODANOV-STOYANOV THEOREM ON MINIMAL ABELIAN TOPOLOGICAL GROUPS

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A topological group X is defined to have *compact exponent* if for some number $n \in \mathbb{N}$ the set $\{x^n : x \in X\}$ has compact closure in X. Any such number n will be called *a compact exponent* of X.

Our principal result states that a complete Abelian topological group X has compact exponent (equal to $n \in \mathbb{N}$) if and only if for any injective continuous homomorphism $f : X \to Y$ to a topological group Y and every $y \in \overline{f(X)} \subset Y$ there exists a positive number k (equal to n) such that $y^k \in f(X)$.

This result has many interesting implications:

(1) each minimal Abelian topological group is precompact (this is the famous Prodanov-Stoyanov Theorem);
(2) an Abelian topological group is compact if and only if it is complete in each weaker Hausdorff group topology (this resolves a problem of Protasov and Zelenyuk of 2001);
(3) an Abelian topological group X is complete and has compact exponent if and only if it is closed in each Hausdorff paratopological group containing X as a topological subgroup (this confirms a conjecture of Banakh and Ravsky of 2001).

More details can be found in the preprint (https://arxiv.org/abs/1706.05411).

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