Lower semicontinuity and tensor products of complete lattices

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The category of all complete lattices and their join preserving maps is well known to have tensor products (the tensor product $M \otimes L$ of complete lattices M and L is by definition the codomain of a universal bimorphism from $M \times L$ into $M \otimes L$). Let X be a topological space with topology $\mathcal{O}(X)$. Let $\mathrm{Lsc}(X, L)$ be the complete lattice of all lower semicontinuous maps f from X into L, i.e. $f(x) = \bigvee \{\bigwedge f(U) : U$ is an open nbhd of $x\}$ for $x \in X$. This talk exhibits conditions on L (or on X) that imply that

$$\mathrm{Lsc}(X, L) \cong \mathcal{O}(X) \otimes L.$$

For instance, this is the case for any X and any continuous lattice L. In general, our lower semicontinuity does not require any topology on L. When L is meet-continuous, we have the following characterization: lower semicontinuity coincides with continuity with respect to a topology τ on L if and only if L is a continuous lattice and τ is the Scott topology of L.

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