IDEAL CONVERGENCE OF SEQUENCES OF QUASI-CONTINUOUS FUNCTIONS

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Let $\mathcal{I}$ be an ideal on ω and let X be a uncountable Polish space. For a given family $\mathcal{F} \subset \mathbb{R}^X$ let $\mathrm{LIM}(\mathcal{F})$ ($\mathcal{I}-\mathrm{LIM}(\mathcal{F})$) denote the class of all limits ($\mathcal{I}$-limits) of sequences of functions from $\mathcal{F}$. Laczkovich and Recław [5] characterized Borel ideals $\mathcal{I}$ for which the class of $\mathcal{I}$-limits of sequences of continuous functions $f : X \to \mathbb{R}$ coincides with the first Baire class. Independently an analogous result has been obtained by Debs and Saint Raymond [2] for analytic ideals, and by Bouziad [1] in a general case (for all ideals).

In [6] we study an analogous problem for the family of all *quasi-continuous* functions. A set $A \subset X$ is *semi-open* if $A \subset \mathrm{cl}(\mathrm{int}(A))$. A function $f : X \to \mathbb{R}$ is quasi-continuous ($f \in \mathrm{QC}(X)$) if $f^{-1}(U)$ is semi-open for any open set $U \subset \mathbb{R}$. A function f is *pointwise discontinuous* ($f \in \mathrm{PWD}(X)$) if set of continuity points of f is dense in X. Finally, $\mathrm{Baire}(X)$ denotes the class of all functions $f : X \to \mathbb{R}$ possessing the Baire property.

Let X be a metric Baire space. Grande [3] proved that

- $\mathrm{LIM}(\mathrm{QC}(X)) = \mathrm{PWD}(X)$;
- $\mathrm{LIM}(\mathrm{PWD}(X)) = \mathrm{Baire}(X)$.

We have proven the following ideal versions of the above equalities. Let $\mathcal{I}$ be a Borel ideal. Then

- $\mathcal{I}-\mathrm{LIM}(\mathrm{Baire}(X)) = \mathrm{Baire}(X)$.

Moreover, if $\mathcal{I}$ is weakly Ramsey, then

- $\mathcal{I}-\mathrm{LIM}(\mathrm{QC}(X)) = \mathrm{PWD}(X)$;
- $\mathcal{I}-\mathrm{LIM}(\mathrm{PWD}(X)) = \mathrm{Baire}(X)$.

But if $\mathcal{I}$ is not weakly Ramsey then we have

- $\mathcal{I}-\mathrm{LIM}(\mathrm{QC}(X)) = \mathrm{Baire}(X)$.

References

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