Large deviation control arising from downside risk minimization against a benchmark

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Supposing that X_t is the diffusion process governed by the stochastic differential equation

$$dX_t = \lambda(X_t)dW_t + \beta(X_t)dt \tag{1.1}$$

we consider the following semi-martingale including control parameter h_t

$$Y_T(h) = \int_0^T g(X_s, h_s)ds + \int_0^T \varphi(X_s, h_s)^* dW_s. \tag{1.2}$$

Then, assuming that $g(x,h)$ is a quadratic function of h defined by $g(x,h) = -\frac{1}{2}h^*S(x)h + h^*g_1(x) + g_0(x)$ and $\varphi(x,h) = F(x)h + f(x)$ we look at asymptotic behavior of the minimizing probability

$$\inf_{h.} P(\frac{1}{T}Y_T(h) \le \kappa) \sim e^{-TI_0(\kappa)}. \tag{1.3}$$

If we do not have the terms depending on control parameter h nor the martingale part in $Y_T(h)$, it is nothing but the occupation time $\int_0^T g_0(X_s)ds$ of diffusion process X_t and far reaching studies have been developed so far concerning its large deviation estimates. Thus, seeing asymptotic behavior of the minimizing probability (1.3) is considered a study of generalization of the large deviation principle for diffusion processes. In this view point, the problems to explore may be formulated as follows.

1. to characterize the rate function $I_0(\kappa)$ in (1.3)
2. to give information about "effective domain" of $I_0(\kappa)$ where $0 < I_0(\kappa) < \infty$
3. to find (asymptotically) an optimal strategy $\hat{h}_t$ which realizes the asymptotic behavior in (1.3)

Such kinds of problems appear in mathematical finance as downside risk minimization for the growth rate of the wealth process in comparison with the preset benchmark process. We actually consider minimizing the probability of falling below a given target growth rate of the semi-martingale functionals given by the log ratio of the total wealth to the benchmark process on a finite time horizon T, and then we look at its exponential decay rate as $T \to \infty$. Establishing the duality relationship between this asymptotic behavior and a certain risk-sensitive control problem over large time, and discussing about the "effective domain" of the rate function of the asymptotics are considered the problems on large deviation control. We present some results obtained about these problems.

We also consider such large deviation control for the quadratic semi-martingale functionals under model uncertainty. Formulating a penalized version of the above problems concerning asymptotic behavior of minimizing the worst case probability of falling below a given target growth rate of the controlled functionals, we address the duality relationship between this penalized problem and a certain risk-sensitive game problem. We then discuss about the "effective domain" of the rate function of the asymptotics as well.