Regress Later Monte Carlo for Optimal Control of Markov Chains

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A common approach in numerical solution of optimal control of discrete-time Markov processes is to discretise the process as a finite state space controlled Markov chain. This method suffers from the course of dimensionality and requires complex methods for design of efficient discretisation. On the contrary, Monte Carlo methods are not affected by the dimensionality of the state space. A successful adaptation of this approach to pricing of American option was performed in [4] and sparked a succession of papers. However, there it was crucial that the control (stopping) does not affect the dynamics of the underlying process. Adaptation of this approach to the problem of optimal control of discrete-time Markov processes was only achieved in [3], where the control was treated as an additional variable in the state space (control randomisation), but the convergence of the proposed scheme was not proved.

In this talk, I will present our approach in which we do not use control randomization. The key to our solution is the regress later approach developed to improve accuracy of American option pricing [2]. It consists of projecting the value function $V(t+1, x)$ over the linear space generated by basis functions $\{\phi_k(t+1, x)\}_{k=1}^K$ and then computing, possibly analytically,

$$\mathbb{E}[V(t+1, X_{t+1})|X_t = x, u_t = u] \approx \sum_{k=1}^{K} \alpha_k^{t+1} \mathbb{E}[\phi_k(t+1, X_{t+1})|X_t = x, u_t = u],$$

where u_t is the control at time t. The regress later approach allows us to: avoid randomisation of the control improving speed and accuracy; place training points freely leading to faster convergence; use performance iteration without resimulation saving computational time. It also allows us to prove that our numerical scheme converges, the task that seems very difficult for the control randomisation method [3].

In the talk I will introduce our approach, explain its links to existing solutions, sketch the convergence result and provide numerical evaluation. This talk extends our results in [1]. This is a joint work with Alessandro Balata.

References

[1] A. Balata and J. Palczewski. Regress-later Monte Carlo for optimal inventory control with applications in energy. *arXiv:1703.06461*, 2017.

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