Probabilistic approach to semilinear equations with Dirichlet operator and Borel measure data

Tomasz Klimsiak, IMPAN

Let E be a separable locally compact metric space and let m be a Radon measure on E with full support. We study existence and nonexistence results for the semilinear equations of the form

$$-Au = f(\cdot, u) + \mu,$$

where $(A, D(A)$ is a self-adjoint Dirichlet operator on $L^2(E; m)$, $f : E \times \mathbf{R} \mapsto \mathbf{R}$ is a measurable function which is continuous and nonincreasing with respect to the second variable and μ is a Borel measure on E. First we present the probabilistic approach to the definition of solutions of the equation and develope the theory of good and reduced measures introduced by H. Brezis, M. Marcus and A.C. Ponce in [1] for the equations of above type with $A = \Delta$. Next we prove Kato's inequality and the inverse maximum principle for solutions of the equation and give a complete characterization of the class of measures for which exits a solution in the case of polynomial growth of f with respect to the second variable.

[1] Brezis, H., Marcus, M., Ponce, A.C.: Nonlinear elliptic equations with measures revisited. In: Mathematical Aspects of Nonlinear Dispersive Equations (J. Bourgain, C. Kenig, S. Klainerman, eds.), Annals of Mathematics Studies, **163**, Princeton University Press, Princeton, NJ, 55–110 (2007)