Hydrodynamic limits for chains of oscillators and Wigner distributions

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In a collaborative work with T. Komorowski and S. Olla, we study the macroscopic behavior of a chain of N coupled harmonic oscillators. In order to provide the system with good ergodic properties, we perturb the Hamiltonian dynamics with random flips of velocities, in such a way as to conserve the energy of particles, and such that momentum conservation is no longer valid. We prove that in a diffusive space-time scaling limit the profiles corresponding to the two conserved quantities converge to the solution of a diffusive system of differential equations. While the elongation follows a simple autonomous linear diffusive equation, the evolution of the energy depends on the gradient of the square of the elongation.
We follow an approach based on Wigner distributions, which permit to control the energy distribution over various frequency modes and provide a natural separation between mechanical and thermal energies. In the macroscopic limit we prove that locally the thermal energy spectrum has a constant density equal to the local thermal energy (or temperature), i.e. that the system is, at macroscopic positive times, in local equilibrium, even though it is not at initial time.