Systems of reflected BSDEs with oblique reflection

Mateusz Topolewski, Nicolaus Copernicus University

Let $U = (U^1, U^2, \ldots, U^N)$ be an N–dimensional progressively measurable (with respect to some filtration F) càdlàg process on $[0, T]$. Let us consider a system of reflected BSDEs defined by

$$Y_t^j = \xi^j + \int_t^T f^j(s, Y_s)ds + \int_t^T dK_s^j - \int_t^T dA_s^j - \int_t^T dM_s^j, \quad t \in [0, T],$$

$$\int_0^T (Y_{t-}^j - H_{t-}^j(Y_{t-}))dK_t^j = \int_0^T (U_{t-}^j - Y_{t-}^j)dA_t^j = 0,$$

$$H_t^j(Y_t) \leq Y_t^j \leq U_t^j, \qquad t \in [0, T]$$

for $j = 1, 2, \ldots, N$. In the equation the random vector $\xi = (\xi^1, \xi^2, \ldots, \xi^N)$ is a given terminal condition, $f = (f^1, f^2, \ldots, f^N)$ is a given function (the generator of the equation), and $H = (H^1, H^2, \ldots, H^N)$ is a function which drives the oblique reflection. We say that a quadruple $(Y, M, K, A) = \{(Y^j, M^j, K^j, A^j)\}_{j=1,2\ldots N}$ of càdlàg processes is a solution of the system if Y is a process of Doob's class D, M is a local martingale such that $M_0 = 0$, K, A are predictable increasing processes with $K_0 = A_0 = 0$, and the above equation is satisfied almost surely.

This problem was investigated by Tang et al. [2] in special case when F is a Brownian filtration and the barrier U is continuous. They showed the existence and uniqueness of a solution with Lipschitz continuous generator f and oblique function H of the form

$$H_t^j(y) = \max_{k \neq j} h_{j,k}(t, y^k), \qquad j = 1, 2, \ldots, N,$$

where $\{h_{j,k}\}_{j,k=1\ldots N}$ are some continuous functions such that $h_{j,k}(t, y) \leq y$ for $y \in R$. On the other hand, Klimsiak [1] considered system of equations without upper barrier U on the general filtered space. He proved the existence result for a quasi–monotone generator f and H being an increasing continuous function. He also showed that if H is given by the above, then the solution is unique.

In the talk we will show that the results of [2] can be generalized to the setting considered in [1]. Moreover, we will present an application of our result to the general optimal switching problem for one-dimensional reflected BSDE, where the generator, the terminal value and the upper barrier are all switched with positive costs.

References:

[1] T. Klimsiak, Systems of quasi-variational inequalities related to the switching problem, arXiv:1609.02221 v2 (2016).

[2] S. Tang, W. Zhong, H. K. Koo, Optimal switching of one-dimensional reflected BSDEs and associated multidimensional BSDEs with oblique reflection, *SIAM J. Control Optim.*, 49 (2011), 2279–2317.