Warsaw Center of Mathematics and Computer Science

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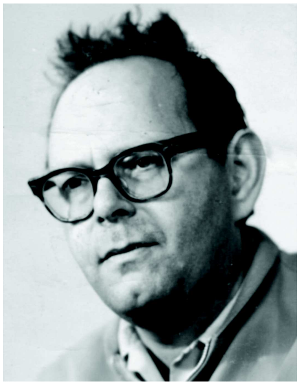
INSTITUTE OF MATHEMATICS PAS

# V Spring School of Analysis



## in memory of

## Aleksander Pełczyński

## Będlewo 30.03-02.04.2017

**Giovanni S. Alberti** (Genova)
Harmonic Analysis of Wavelets
and detection of singularities

**Bernd Kirchheim** (Leipzig)
Lower Semicontinuouity
in vectorial variational problems

**Xavier Tolsa** (Barcelona)
The Riesz Transform rectifiability
and harmonic measure

Organizer: Michał Wojciechowski (IMPAN)
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## Harmonic analysis of wavelets and detection of singularities

### Giovanni S. Alberti

A fundamental tool in signal analysis is represented by wavelets, and by their many multi-dimensional generalisations, such as shearlets. The main focus of this course is the abstract harmonic analysis of wavelets and shearlets. After a brief review of unitary representations, we shall first discuss the one-dimensional case, and in particular the characterization of mother wavelets and the detection of discontinuities. We will then treat the higher dimensional case, with a particular focus on the resolution of the wavefront sets achieved with shearlets.

## Lower semicontinuity in vectorial variational problems

### Bernd Kirchheim

The direct method of the Calculus of Varitions plays an irreplaceable role when minimizers are needed. For scalar valued maps (functions) the crucial weak lower semicontinuity is essentially characterized by convexity of the energy integrand with respect to the derivative, and quite well understood. In the case of vectorial problems, however, there is no equally efficient, and in particular geometric condition known. Instead a whole zoo of necessary or equivalent (but not really understood) or sufficient conditions had to be introduced, and will be discussed in the course.

It is well know by now, that all these semiconvexity notions differ in general. We will however, describe some fairly general results when and to which extend these notions agree and also discuss how minimizers of variational problems can (sometimes) be found in the absence of lower semicontinuity.

## The Riesz transform, rectifiabilty, and harmonic measure

### Xavier Tolsa

These lectures will be devoted to the David-Semmes problem and its applications to the study of the geometric properties of harmonic measure. Roughly speaking, the David-Semmes problem consists in proving that, given a set $E \subset \mathbb{R}^d$ with finite Hausdorff $n$-dimensional measure $H^n$, the $L^2$ boundedness of the $n$-dimensional Riesz transform with respect to the Hausdorff measure $H^n|_E$ implies the $n$-rectifiability of $E$. In the late 1990's, in the case $n = 1$ this problem was solved for AD-regular sets by Mattila, Melnikov and Verdera, and in full generality by David and Léger, by using the connection between Menger curvature and the Cauchy kernel. The case of codimension 1 ($n = d - 1$) was solved more recently by Nazarov, Tolsa and Volberg, by combining quasiorthogonality and variational arguments. The David-Semmes problem is still open for $n$ different from 1 and $d - 1$.

The second part of the lectures will deal the applications to the study of harmonic measure and rectifiability. In particular, we will see a recent result by Azzam, Hofmann, Martell, Mayboroda, Mourgoglou, Tolsa and Volberg which asserts that if the harmonic measure is absolutely continuous with respect to the Hausdorff measure $H^n$ on some subset $F$ of the boundary of an open set in $\mathbb{R}^{n+1}$, with $H^n(F) < \infty$, then $F$ is $n$-rectifiable. This result can be considered as a converse of the famous theorem of the Riesz brothers on harmonic measure for simply connected domains in the plane, with no topological assumptions in $\mathbb{R}^{n+1}$.

Further, we will also review another recent application to a two phase problem for harmonic measure by Azzam, Mourgoglou and Tolsa, whose proof uses the connection between Riesz transforms and rectifiability and some blowup techniques inspired by work of Kenig, Preiss and Toro. If time permits, we will see an easy application of these blowup methods to the proof of Tsirelson's theorem about triple points for harmonic measure.

Conference Cost: 100 EUR

For registration and information
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