From the conformal group to symmetries of hypergeometric type equations.

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Abstract.

I. First I will explain how the usual Lie algebra $\mathbb{R}^n \rtimes so(n)$ of symmetries of the Euclidean space extends to the Lie algebra of conformal symmetries $so(n+1,1)$. In a certain generalized sense, the Laplace equation turns out to be invariant wrt conformal symmetries. I will also describe the closely related Lie algebra $sch(n)$ of generalized symmetries of the heat equation, usually called the Schrödinger Lie algebra.

II. Next I will make an overview of the so-called hypergeometric-type equations. Their solutions include special functions that are widely used in applications such as the hypergeometric function, the confluent function and the Bessel function. I will discuss typical classes of identities satisfied by these functions.

III. Properties of hypergeometric type equations become quite transparent if they are derived from appropriate 2nd order partial differential equations with constant coefficients, that is from the Laplace, heat and Helmholtz equations. These facts can be summarized by the following table:

PDE	Lie algebra	dimension of Cartan algebra	discrete symmetries	equation
Δ_4	$so(6)$	3	cube	hypergeometric;
Δ_3	$so(5)$	2	square	Gegenbauer;
$\Delta_2 + \partial_t$	$sch(2)$	2	$\mathbb{Z}_2 \times \mathbb{Z}_2$	confluent;
$\Delta_1 + \partial_t$	$sch(1)$	1	$\mathbb{Z}_4$	Hermite;
$\Delta_2 - 1$	$\mathbb{R}^2 \rtimes so(2)$	1	$\mathbb{Z}_2$	Bessel.

The lectures are based on [1, 2]

References

[1] Jan Dereziński, Hypergeometric type functions and their symmetries, *Annales Henri Poincare* 15 (2014), 1569-1653.

[2] Jan Dereziński, Przemysław Majewski, From conformal group to symmetries of hypergeometric type equations, SIGMA 12 (2016).