Different aspects of summability

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Abstract

The aim of this course is to provide the main tools to understand the notion of summability of solutions of holomorphic differential equations, in one and in several complex variables. Summability was developed through several works by Wasow, Sibuya, Ramis, Balser, Braaksma, Ecalle and others, in order to give a meaning to formal solutions of holomorphic ordinary differential equations. For more than one complex variable, different notions of asymptotic expansions were introduced by Gérard, Sibuya and Majima. More recently, the notion of monomial summability was defined by Canalis-Durand, Schäfke and the author, in order to treat singularly perturbed differential equations. This notions has been succesfully applied to other kind of functional problems.

In this course we will explain these notions, giving enough examples to show its validity. We will begin by the classical theory and we will focus in the problems that can be treated under monomial summability.

A tentative schedule of the 5 sessions of the course will be as follows:

1. Summation of divergent series. Classical examples. Euler and Airy equations. Glimpses to the Stokes phenomena.
2. Summability in one variable. Borel and Laplace transforms. Applications.
3. Monomial summability. Definitions. Relation with one variable summability.
4. Tauberian theorems. Behaviour under blow-ups. Notice about summability with respect to an analytic germ.
5. Applications. Singularly perturbed differential equations. Pfaffian systems. Normalization of vector fields.