Movable singularity of some Hamiltonian system and blowup of semilinear wave equation

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Abstract

The motivation of this lecture comes from the blow-up phenomenon for the semilinear wave equation

$$U_{tt} - \Delta U - U^\ell = 0, \; U = U(x,t), \; x = (x_1, \ldots, x_n) \in \mathbb{R}^n, t \in \mathbb{R}, \qquad (1)$$

where $n \geq 2$ and ℓ is an integer. One often considers the self-similar radially symmetric solution $U := (T - t)^{-2/(\ell-1)} u(r/(T - t))$, where $T > 0$, $r^2 = x_1^2 + \cdots + x_n^2$, and $u \equiv u(y)$ is a function of one variable. u satisfies the semi linear Heun equation $(1 - y^2)u'' + ((n-1)/y + ay) u' + bu + u^\ell = 0$ for some constants a and b. The equation is written in a Hamiltonian system for which we construct a singular solution via the theory of dynamical systems. Applying the Birkhoff normal form theory, we see that the normalizing symplectic transformation is divergent due to the non integrability or the blow-up phenomenon of the original problem. We treat the divergence by the monomial summability theory. Then we construct the blow-up solution with singularities on the characteristic cone. We also show the weak Painlevé property for second order ordinary differential equation with polynomial nonlinearity and polynomial coefficients by the analytic continuation of the Borel sum. We also treat non integrability and further extensions. Our plan of the lecture is as follows.

1. Blow-up of a semi linear wave equation and a semi linear Heun equation as a profile equation.

2. Formal Birkhoff reduction divergent with respect to a certain monomial.

3. Monomial summability of divergent Birkhoff transformation.

4. Construction of movable singularities and the blow-up of the semi linear wave equation with singularities on the characteristic cone.

5. Weak Painlevé property for a second order ordinary differential equation with polynomial coefficients and polynomial nonlinearity. – analytic continuation of the Borel sum of Birkhoff transformation –

6. Non integrability and further extensions

References

[1] Yoshino, M., Movable singularity of generalized Emden equation via Birkoff reduction, *To be published in RIMS Kôkyûroku Bessatsu.*

[2] Yoshino, M., Movable Singularity of Hamiltonian System and Blowup of Semi linear Wave Equation, *Preprint.*