Maillet Type Theorem for Nonlinear Totally Characteristic Partial Differential Equations

Hidetoshi TAHARA
(Sophia University, Tokyo, JAPAN)

Let us consider nonlinear singular partial differential equations

$$(t\partial_t)^m u = F\big(t, x, \{(t\partial_t)^j \partial_x^\alpha u\}_{j+|\alpha|\le m, j<m}\big) \tag{E}$$

in the complex domain under the assumption that $F(t,x,z)$ (with $z = \{z_{j,\alpha}\}_{j+|\alpha|\le m, j<m}$) is a holomorphic function satisfying $F(0,x,0) \equiv 0$. This equation is classified into the following three types:

Type 1. $(\partial F/\partial z_{j,\alpha})(0,x,0) \equiv 0$ for all (j,α) with $|\alpha| > 0$
Type 2. $(\partial F/\partial z_{j,\alpha})(0,0,0) \neq 0$ for some (j,α) with $|\alpha| > 0$.
Type 3. the other case.

Usually, the equation of type 3 is called a nonlinear totally characteristic type partial differential equation. In this talk, We mainly treat equations of type 3.

We define a non-resonance condition (N), Newton polygon of the equation at $x = 0$, and the generalized Poincaré condition (GP). Then, by using the Newton Polygon, the notion of the irregularity at $x = 0$ of the equation is defined. In the case where the irregularity is greater than one, it is proved under (N) and (GP) that (E) has a unique formal power series solution and it belongs to a suitable formal Gevrey class. The precise bound of the order of the formal Gevrey class is given, and the optimality of this bound is also proved in a generic case.

The result gives a lot of examples of equations which has such a formal power series solution that it is divergent but we don't know whether it is summable or not.

This is a continuation of my talk in FASdiff17 (Alcala, 2017), and is based on a joint work with A. Lastra (University of Alcala).