Intersection theory for Euler integral representations of GKZ hypergeometric functions

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The integration of the product of powers of polynomials has a long history dating back to Aomoto's pioneering work for hypergeometric functions on Grassman varieties. We call such an integral Euler integral. It was later clarified by Gelfand, Kapranov, and Zelevinsky that the Gauss-Manin system satisfied by Euler integral is nothing but GKZ hypergeometric system when parameters are non-resonant. Though GKZ system has rich combinatorial structures such as regular triangulations and secondary polytopes, it has been unclear that how such structures are translated in the language of twisted cycles. In this talk, we relate the combinatorial structure of GKZ system to the explicit construction of twisted cycles. It turns out that the intersection form of the twisted homology groups is naturally block-diagonalized with respect to such a basis of cycles. When the regular triangulation is unimodular, we can obtain a closed formula for the intersection numbers. Time permitting, we give a general twisted period relation formula for unimodular triangulations which gives rise to some new quadratic relations for hypergeometric series in several variables.

References

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