On the instanton-type expansions for Painlevé transcendents and elliptic functions

Yoshitsugu Takei
Department of Mathematical Sciences
Doshisha University
ytakei@mail.doshisha.ac.jp

As was verified by Kapaev and others, the most basic Stokes phenomena for Painlevé transcendents are described by formal power series solutions and transseries solutions of Painlevé equations. However, to discuss more general Stokes phenomena, we need to deal with instanton-type solutions, which are purely formal and whose behavior are much more wild than transseries solutions. In this talk, from the viewpoint of exact WKB analysis, we investigate instanton-type formal solutions of Painlevé equations and those of the equation for Weierstrass' elliptic functions. After explaining the construction of instanton-type solutions, we discuss how to give an analytic meaning to them. In particular, in the case of the equation for Weierstrass' elliptic functions it turns out that instanton-type solutions are nothing but Fourier expansions of elliptic functions.