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School and Conference

COMPLEX DIFFERENTIAL AND DIFFERENCE EQUATIONS

September 2 – 15, 2018

Mathematical Research and Conference Center, Będlewo

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School **Complex Differential and Difference Equations**

September 3 – 8, 2018

Abstracts of lectures

Jan Dereziński (Faculty of Physics of the Warsaw University, POLAND)

From the conformal group to symmetries of hypergeometric type equations

Properties of hypergeometric type equations become quite transparent if they are derived from appropriate 2nd order partial differential equations with constant coefficients. In particular, symmetries of the hypergeometric and Gegenbauer equation follow from from conformal symmetries of the 4- and 3-dimensional Laplace equation. The symmetries of the confluent and Hermite equation follow from the so-called Schrödinger symmetries of the heat equation in 2 and 1 dimension. Finally, properties of the ${}_0F_1$ equation follow from the Helmholtz equation in 2 dimensions.

The lectures are based on [1, 2].

References

[1] Jan Dereziński, *Hypergeometric type functions and their symmetries*, Annales Henri Poincaré **15** (2014), 1569–1653.

[2] Jan Dereziński, Przemysaw Majewski, *From conformal group to symmetries of hypergeometric type equations*, SIGMA **12** (2016).

Shingo Kamimoto (Hiroshima University, JAPAN)

Resurgent structures in differential equations

The aim of this series of lectures is to study resurgent structures of formal series solutions of differential equations. Resurgent analysis was developed by J. Écalle in his book [1] and now, his theory is widely used in several areas: Dynamical systems, multiple zeta values, Mathematical physics and so on.

In this lecture, we focus on the analysis of differential equations. We start from the basics of resurgent analysis and explain resurgent structures of the formal series solutions from the viewpoint of convolution products based on [2] and [3]. Further, we mention recent trends in resurgent analysis.

A tentative plan of the lecture is the following:

(1) Basics in resurgent analysis
(2) Convolution product and resurgence
(3) Resurgent structure of formal series solutions
(4) Recent trends in resurgent analysis

References

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[2] S. Kamimoto: *Resurgence of formal series solutions of nonlinear differential and difference equations*, Proc. Japan Acad. Ser. A Math. Sci. **92** (2016), no. 8, 92–95.

[3] S. Kamimoto and D. Sauzin: *Iterated convolutions and endless Riemann surfaces* , to appear in Annali della Scuola Normale Superiore di Pisa, Classe di Scienze.

JORGE MOZO FERNÁNDEZ (Universidad de Valladolid, SPAIN)

Different aspects of summability

The aim of this course is to provide the main tools to understand the notion of summability of solutions of holomorphic differential equations, in one and in several complex variables. Summability was developed through several works by Wasow, Sibuya, Ramis, Balser, Braaksma, Écalle and others, in order to give a meaning to formal solutions of holomorphic ordinary differential equations. For more than one complex variable, different notions of asymptotic expansions were introduced by Gérard, Sibuya and Majima. More recently, the notion of monomial summability was defined by Canalis-Durand, Schäfke and the author, in order to treat singularly perturbed differential equations. This notion has been succesfully applied to other kind of functional problems.

In this course we will explain these notions, giving enough examples to show its validity. We will begin by the classical theory and we will focus in the problems that can be treated under monomial summability.

A tentative schedule of the 5 sessions of the course will be as follows:

(1) Summation of divergent series. Classical examples. Euler and Airy equations. Glimpses to the Stokes phenomena.
(2) Summability in one variable. Borel and Laplace transforms. Applications.
(3) Monomial summability. Definitions. Relation with one variable summability.
(4) Tauberian theorems. Behaviour under blow-ups. Notice about summability with respect to an analytic germ.
(5) Applications. Singularly perturbed differential equations. Pfaffian systems. Normalization of vector fields.

JUAN RAFAEL SENDRA (University of Alcalá, SPAIN),
FRANZ WINKLER (J. Kepler University Linz, AUSTRIA)

Algebro-geometric method for algebraic differential equations

An algebraic differential equation (ADE) is given by a polynomial relation between a desired function $y(x_1, \ldots, x_n)$, and some of its derivatives. If $n = 1$ we speak of an algebraic ordinary differential equation (AODE), otherwise of an algebraic partial differential equation (APDE). We are interested in finding prescribed types of solutions, e.g., rational or algebraic solutions, to such ADEs. Indeed, if possible, we determine so-called general solutions, which are whole multiparametric families of solutions. The problem of determining general solutions of first-order AODEs can be dated back to the work of L. Fuchs and H. Poincaré.

In the algebro-geometric method we associate to a given (system of) ADE(s) an algebraic variety. From the rational components of this variety we determine explicit rational solutions of the differential problem; indeed so-called rational general solutions, i.e., full families of rational solutions. As an example consider the AODE $F(x, y, y') \equiv y'^2 + 3y' - 2y - 3x = 0$. A rational general solution is $y(x) = \frac{1}{2}(x^2 + 2cx + c^2 + 3c)$.

The basic idea of this approach was described by Feng and Gao in [1] and [2] for the situation of an autonomous AODE of order 1, $F(y, y') = 0$, F an irreducible bivariate polynomial over $\mathbb{Q}$. If the AODE has a rational solution $y(x)$, then $(y(x), y'(x))$ is a rational parametrization of the so-called associated curve $\mathcal{C} : F(u, v) = 0$. Indeed, as they prove, this is a proper 1-1 parametrization. All proper parametrizations are related by linear rational transformations. So we need to decide whether we can find a transformation which leads to a parametrization, in which the second component is the derivative of the first. This can be done algorithmically, leading to a rational solution of the AODE or to the answer that no such rational solution exists.

The method has been extended to non-autonomous AODEs in [4], and recently to algebraic partial differential equations in [3].

References

[1] R.Feng, X.-S.Gao, “Rational general solutions of algebraic ordinary differential equations”, Proc. ISSAC’2004, 155–162, ACM Press (2004)

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Masafumi Yoshino (Hiroshima University, JAPAN)

Integrability of Hamiltonian systems. Movable singularity of some Hamiltonian system and blowup of semilinear wave equation

The motivation of this lecture comes from the blow-up phenomenon for the semilinear wave equation

$$U_{tt} - \Delta U - U^{\ell} = 0, \quad U = U(x,t), \quad x = (x_1, \ldots, x_n) \in \mathbb{R}^n, t \in \mathbb{R}, \tag{1}$$

where $n \geq 2$ and ℓ is an integer. One often considers the self-similar radially symmetric solution $U := (T-t)^{-2/(\ell-1)} u(r/(T-t))$, where $T > 0$, $r^2 = x_1^2 + \cdots + x_n^2$, and $u \equiv u(y)$ is a function of one variable. u satisfies the semi linear Heun equation $(1-y^2)u'' + ((n-1)/y + ay)\, u' + bu + u^{\ell} = 0$ for some constants a and b. The equation is written in a Hamiltonian system for which we construct a singular solution via the theory of dynamical systems. Applying the Birkhoff normal form theory, we see that the normalizing symplectic transformation is divergent due to the non integrability or the blow-up phenomenon of the original problem. We treat the divergence by the monomial summability theory. Then we construct the blow-up solution with singularities on the characteristic cone. We also show the weak Painlevé property for second order ordinary differential equation with polynomial nonlinearity and polynomial coefficients by the analytic continuation of the Borel sum. We also treat non integrability and further extensions. Our plan of the lecture is as follows.

1. Blow-up of a semi linear wave equation and a semi linear Heun equation as a profile equation.
2. Formal Birkhoff reduction divergent with respect to a certain monomial.
3. Monomial summability of divergent Birkhoff transformation.
4. Construction of movable singularities and the blow-up of the semi linear wave equation with singularities on the characteristic cone.
5. Weak Painlevé property for a second order ordinary differential equation with polynomial coefficients and polynomial nonlinearity. – analytic continuation of the Borel sum of Birkhoff transformation –
6. Non integrability and further extensions

References

[1] Yoshino, M., Movable singularity of generalized Emden equation via Birkoff reduction, *To be published in RIMS Kôkyûroku Bessatsu.*

[2] Yoshino, M., Movable Singularity of Hamiltonian System and Blowup of Semi linear Wave Equation, *Preprint.*

Conference Complex Differential and Difference Equations

September 10 – 15, 2018

Abstracts of lectures and talks

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Tanuja Adaviswamy
Siddaganga Institute of Technology, Tumkur, INDIA

Meromorphic solution of Ginzburg-Landau family of equations with variable coefficients using Nevanlinna theory

Ginzburg-Landau equation (GLE) arise in fluid dynamical problems while performing a weakly nonlinear stability analysis of the system. The solution of such equations is presented here by considering the complex differential equation corresponding to GLE. The existence of a meromorphic solution of the GLE is demonstrated using Nevanlinna theory with sharing value one countable multiplicitiy or ignoring multiplicitiy. Nevanlinna theory has emerged as an efficient tool in the study of complex differential and difference equations.

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Yulia Bibilo
Concordia University, CANADA

Application of the Painlevé 3 equation to the Josephson effect

Following series of articles we consider a family of nonlinear differential equations

$$\frac{d\phi}{dt} = -\sin(\phi) + B + A\cos(\omega t), \tag{2}$$

where A,B, and ω are scalar real parameters. This equation models the Josephson effect in superconductivity. Rotation number is a function of parameters A, B, ω and it gives some description of periodic trajectories of (2); its phase locking domains were studied by Arnold, Ilyashenko, Buchstaber, and Glutsyuk.

We obtained a new approach which gives a description of the phase locking domains and adjacency points via poles of Bessel solution of Painlevé 3 equation.

References

[1] Y. Bibilo, *Josephson Effect and Isomonodromic Deformations*, 2018, https://arxiv.org/abs/1805.11759

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Ewa Ciechanowicz
University of Szczecin, POLAND

Value distribution and growth of solutions of certain nonlinear ODEs

Let $R(z; f)$ be rational in f with meromorphic coefficients. By an extension of the Malmquist-Yosida theorem, if the equation $(f')^n = R(z; f)$ takes up an admissible meromorphic solution, then $R(z; f)$ is a polynomial in f; which means that the equation is the hyper-Riccati equation. As a result of classification of the second order ordinary differential equations without movable branch points, $f'' = F(z; f; f')$; where F is rational in f, algebraic in f' and analytic in z, so-called Painlevé equations have been obtained. Among them, six irreducible equations are the best known. They led to recognition of new functions, called the Painlevé transcendents. The Painlevé equations possess a number of remarkable properties, the Hamiltonian structure in particular. By this structure they are related both with one another and with a number of associated equations, called Painlevé sigma-equations. Meromorphic solutions of Riccati, hyper-Riccati and Painlevé equations can be studied from the perspective of value distribution and growth theory, with such values as defect, deviation or multiplicity index estimated.

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Galina Filipuk (a joint work with M. N. Rebocho)
University of Warsaw, POLAND

Orthogonal polynomials on non-uniform lattices

I shall briefly discuss orthogonal polynomials on non-uniform lattices and show that the three-term recurrence coefficients satisfy a system of nonlinear difference equations. This is a joint work with M. N. Rebocho (Portugal).

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Pilar Ruiz Gordoa
Universidad Rey Juan Carlos, Madrid, SPAIN

Matrix Painlevé hierarchies

In this talk we consider the construction of hierarchies of matrix ordinary differential equations, analogous to scalar Painlevé hierarchies. By considering generalized matrix KdV and mKdV hierarchies we derive a matrix first Painlevé hierarchy and a matrix second Painlevé hierarchy. The relationship between the matrix mKdV equation and the matrix second Painlevé equation is thus clarified. We also investigate properties of the matrix second Painlevé hierarchy, e.g., auto-Bäcklund transformations. We consider further examples of matrix Painlevé equations. Our work shows how properties of matrix ODEs can be derived using structures of related matrix PDEs.

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Hubert Grzebuła (a joint work with S. Michalik)
Cardinal Stefan Wyszyński University, Warsaw, POLAND

The Dirichlet type problem for complex polyharmonic functions

In this talk we consider some Dirichlet-type problem for polyharmonic functions where the boundary conditions are given only in the terms of the solution. Here the conditions on the normal derivatives or on the iterated Laplacians of the solution are replaced by the conditions given in terms of values of the solution on rotated spheres.

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Davide Guzzetti
SISSA, Trieste, ITALY

Non-generic isomonodromy deformations

Some of the main results of [1] (see also [3] for a synthetic exposition with examples), concerning non-generic isomonodromy deformations of a certain linear differential system with irregular singularity and coalescing eigenvalues, are discussed from the point of view of Pfaffian systems adopted in [5], making a distinction between weak and strong isomonodromic deformations. The results are motivated by the problem of extending to coalescent structures the analytic theory of Frobenius manifolds [2], [3], [6].

References

[1] G. Cotti, B. Dubrovin. D. Guzzetti: Isomonodromy Deformations at an Irregular Singularity with Coalescing Eigenvalues. arXiv:1706.04808 (2017).
[2] G. Cotti, B. Dubrovin. D. Guzzetti: Local Moduli of Semisimple Frobenius Coalescent Structures. arXiv:1712.08575 (2017).
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[5] D. Guzzetti: Notes on non-generic Isomonodromy Deformations. arXiv:1804.05688 (2018)
[6] G. Cotti et al. Helix Structures in Quantum Cohomology of Fano Varieties. To appear

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Yoshishige Haraoka
Kumamoto University, JAPAN

Asymptotic analysis for a confluent KZ equation in two variables

We are interested in asymptotic analysis of completely integrable systems of irregular singular type in several variables. Majima (LNM 1075, Springer, 1984) gave a fundamental idea of asymptotic expansion in several variables, and developed a general theory. Shimomura (Proc. Royal Soc. Edinbrgh, 123 (1993), 1165-1177; J. Math. Anal. Appl., 187 (1994), 468-484) studied asymptotic behaviors of some confluent hypergeometric functions in two variables in a different way from Majima's asymptotics. The asymptotic analysis in several variables seems to be difficult because there were few examples of completely integrable systems. In applying the Katz theory on rigid local systems, we get a way of constructing completely integrable systems in a recursive way, and can obtain infinitely many examples. These examples will be helpful to develop a general theory. In this talk, we study a confluent KZ equation of rank four in two variables (x, y) with irregular singularity at $x = \infty$ and $y = \infty$. We will analyze these singularities in Shimomura's way and in Majima's way.

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Kunio Ichinobe
Aichi University of Education, JAPAN

A necessary condition for k-summability of formal solutions to some linear q-difference-differential equations

We study k-summability of the formal solutions to the Cauchy problem for linear q-difference-differential equations which are kind of q-analog of linear partial differential equations of non-Kowalevski type as the heat equation. In this talk, we give a necessary and sufficient condition for the k-summability of formal solutions in terms of a global analyticity and a exponential growth estimate of the Cauchy data, whose conditions are a little different from the ones for PDEs.

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Hideaki Izumi
Chiba Institute of Technology, JAPAN

Dimensioned numbers and differential equations

We consider numbers with dimensions: a_b $(a, b \in \mathbb{R})$ represents the quantity a of dimension b. This kind of numbers should satisfy the following law:

$$a_b + a'_b = (a + a')_b,\ \ \mathrm{a_b \cdot c_d = ac_{b+d}}.$$

This is quite similar to the computation of monomials:

$$ax^b + a'x^b = (a + a')x^b,\ \ \mathrm{ax^b \cdot cx^d = acx^{b+d}}.$$

Hence the set of the finite sums of numbers of the form a_b $(\mathrm{a} \in \mathbb{R},\ \mathrm{b} \in \mathbb{N})$ form a model for the polynomial ring $\mathbb{R}[x]$. Moreover, we also consider the case where the dimension itself has its dimension. Hence we may consider *dimensioned numbers* ([1]) of any depth:

$$2_{3_4} \sim 2x^{3x^4},\ \ 5_{-3_4+2_{-6_1}} \sim 5\mathrm{x}^{-3\mathrm{x}^4+2\mathrm{x}^{-6\mathrm{x}}}.$$

In general, for any dimensioned number A, which represents a function $A(x)$, p_A $(\mathrm{p} \in \mathbb{R})$ represents $px^{A(x)}$. Conversely, if $B = 1_A$ represents $B(x)$, then A represents $\log_x B(x)$. We develop differential operations on dimensioned numbers and obtain formal solutions of ordinary differential equations which involve algebraic, exponential and logarithmic operations.

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Shingo Kamimoto (a joint work with J. Jiménez-Garrido, A. Lastra and J. Sanz)
Hiroshima University, JAPAN

Multisummability and strongly regular sequences

The theory of multisummable series was developed by J.-P. Ramis, J. Écalle, W. Balser and other mathematicians. Multisummable series naturally appear in the analysis of differential equations. It is useful to analyze the structure of irregular singular points of differential equations. There are several approaches to the multisummability; via acceleration operators, via decomposition of multisummable series to summable series, via quasi-functions.

On the other hand, the summability method for strongly regular sequences is organized by J. Sanz, S. Malek, A. Lastra and J. Jiménez-Garrido. In this talk, we explain the theory of multisummable series for strongly regular sequences based on their work and make clear the algebraic frame work of the theory of multisummability.

This work is based on the joint work with Javier Jiménez-Garrido, Alberto Lastra and Javier Sanz.

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Thomas Kecker (partially a joint work with G. Filipuk)
University of Portsmouth, UNITED KINGDOM

Complex differential equations with movable algebraic singularities

A classical question in the theory of ordinary differential equations in the complex plane is to determine what types of movable singularities (i.e. those singularities depending on the initial data of the equation) a local, analytic solution can develop when analytically continued in the complex plane. For certain classes of second-order differential equations and two-dimensional Hamiltonian systems of equations it is shown that under certain mild conditions, the only movable singularities that can occur in a solution are of algebraic type, represented by Puiseux series with finite principle parts [1]. In the special case that all movable singularities are poles one obtains instances of the Painlevé equations, certain integrable non-linear second- order differential equations which possess a rich mathematical structure and the solutions of which, the so-called Painlevé transcendents, are also referred to as non-linear special functions. A Hamiltonian system of this type is also studied in [2]. One particularly interesting aspect of the Painlevé equations is the so- called Okamoto's space of initial conditions, which is obtained by compactifying the phase space on which the original equation is defined to some rational surface, followed by a number of blow-ups, a construction to remove certain points at which the equation is of indeterminate form. In the case of the Painlevé equations, the space is uniformly foliated by the solutions of the equation and around every point in this space the equation is defined as a regular initial value problem, see also [3] for the system introduced in [2]. The concept of the space of initial conditions can be extended to certain classes of differential equations with algebraic singularities mentioned above, leading to an algorithmic procedure by which to determine, for a given equation, what different types of movable singularities can occur in the solutions of the equations in the complex plane. This part of the talk is joint work with Galina Filipuk.

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[3] T. Kecker (2018), Space of initial conditions for a cubic Hamiltonian system, Complex Variables and Elliptic Equations, 11 pages (published online).

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Piotr Kokocki
Nicolaus Copernicus University, Toruń, POLAND

The Riemann-Hilbert problem and singularity formation in the localized induction approximation

We are concern with the differential equation of the form

$$z_t = -k_s n - \frac{1}{2}k^2 T, \quad t, s \in \mathbb{R}, \tag{3}$$

where z is the flow of regular curves living in the complex plane, s is the arc-length parameter, T is the field of tangent vectors, $n := iT$ is the oriented normal vector field and k is the curvature defined by $T_s = kn$. The equation (3) is called the *localized induction approximation* and can be considered as the geometric flow, whose evolution is similar to the contour dynamics of a vortex patch subjected to the 2D Euler equation (see e.g. [3]). Our aim is to show the existence of a regular family of self-similar solutions for (3), which develops a spiral singularity at finite time. To be more precise, we prove that for any $a \in (-\frac{\pi}{2}, \frac{\pi}{2})$ and $\mu \in \mathbb{R}$, there are $\theta^+, \theta^- \in [0, 2\pi)$ and a self-similar solution $z^{\mu,a}$ of the equation (3), such that $\theta^+ - \theta^- = 2a$ and $\lim_{t\to 0} z^{\mu,a}(t) = z_0^{\mu,a}$ in the space of tempered distributions, where

$$z_0^{\mu,a}(s) = \frac{s}{\sqrt{1+4\mu^2}} e^{i(\theta^- - 2\mu \log|s|)} \chi_{(-\infty,0)}(s) + \frac{s}{\sqrt{1+4\mu^2}} e^{i(\theta^+ - 2\mu \log s)} \chi_{(0,+\infty)}(s), s \in \mathbb{R}. \tag{4}$$

Furthermore we show that the curvature flow $k^{\mu,a}$ corresponding to the solution $z^{\mu,a}$ satisfies the modified Korteweg-de Vries (mKdV) equation

$$k_t + k_{sss} + \frac{3}{2}k^2 k_s = 0 \tag{5}$$

with the following initial condition

$$k^{\mu,a}(s,0) = a\,\delta(s) + \mu\,\mathrm{p.v.}\,(1/s), \tag{6}$$

where $\delta(s)$ is the Dirac delta function and p.v. $(1/s)$ is the Cauchy principal value. The function (4) is called a *logarithmic spiral* and plays a crucial role in the fluid mechanics and turbulence modeling. The method of the proof is searching for the solutions $z^{\mu,a}$ in a class of self-similar functions, whose profiles are purely imaginary solutions of the second Painlevé (PII) equation

$$u''(x) = xu(x) + 2u^3(x) - \alpha \tag{7}$$

with $\alpha := -i\mu$. Following [2], each solution of the (PII) equation is determined by a Riemann-Hilbert problem whose jump function is expressed by Stokes multipliers $(s_1, s_2, s_3) \in \mathbb{C}^3$ satisfying the constraint

$$s_1 - s_2 + s_3 + s_1 s_2 s_3 = -2\sin(\pi\alpha).$$

After improving the integral formulas from [1], we find $m \in i\mathbb{R}$ such that the purely imaginary Ablowitz-Segur solution corresponding to the following choice of the Stokes data

$$s_1 = -\sin(i\pi\mu) - im, \quad s_2 = 0, \quad s_3 = -\sin(i\pi\mu) + im, \tag{8}$$

is an appropriate profile in the construction of the solution $z^{\mu,a}$. The obtained results improves those from [5], where the authors use the perturbation argument together with the methods of ODEs to prove the existence of solutions $z^{\mu,a}$ for the parameters μ and a that are sufficiently close to zero.

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Grzegorz Łysik
Jan Kochanowski University in Kielce, POLAND

Mean values and heat type equations

We shall give a survey of results on convergence and Borel summability of formal power series solutions of some heat type differential equations. Mainly we shall concentrate on equations of the form

$$\partial_t u(t,z) = P(\partial_z) u(t,z),$$

where P is an elliptic differential operator.
The necessary and sufficient conditions for convergence and summability of formal solutions will be given in terms of integral means over balls of the initial data.

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Stephane Malek (a joint work with A. Lastra
University of Lille, FRANCE

On parametric Borel summability for linear singularly perturbed Cauchy problems with linear fractional transforms

We consider a family of linear singularly perturbed Cauchy problems which combines partial differential operators and linear fractional transforms. We construct a collection of holomorphic solutions on a full covering by sectors of a neighborhood of the origin in $\mathbb{C}$ with respect to the perturbation parameter. This set is built up through classical and special Laplace transforms along piecewise linear paths of functions which possess exponential or super exponential growth/decay on horizontal strips. A fine structure which entails two levels of Gevrey asymptotics of order 1 and so-called order 1^+ is witnessed. Furthermore, unicity properties regarding the 1^+ asymptotic layer are observed and follow from results on summability w.r.t a particular strongly regular sequence recently obtained in a work by A. Lastra and J. Sanz. (Joint work with Alberto Lastra).

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Toshiyuki Mano (a joint work with M. Kato and J. Sekiguchi)
University of the Ryukyus, Okinawa, JAPAN

Analytic representation of potential vector fields and isomonodromic τ functions

In a joint work with M. Kato and J. Sekiguchi, we introduced flat coordinates and potential vector fields on the space of variables of isomonodromic deformations of an Okubo system. A potential vector field satisfies the extended WDVV equation, which is a generalization of the fact that a prepotential of a Frobenius manifold is a solution to the WDVV equation.

In this talk, we study analytic solutions to the (extended) WDVV equation. As its consequence, we find an analytic representation of τ functions attached to solutions to the sixth Painlevé equation in terms of flat coordinates.

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Saiei-Jaeyeong Matsubara-Heo
Kobe University, JAPAN

Intersection theory for Euler integral representations of GKZ hypergeometric functions

The integration of the product of powers of polynomials has a long history dating back to Aomoto's pioneering work for hypergeometric functions on Grassman varieties. We call such an integral Euler integral. It was later clarified by Gelfand, Kapranov, and Zelevinsky that the Gauss-Manin system satisfied by Euler integral is nothing but GKZ hypergeometric system when parameters are non-resonant. Though GKZ system has rich combinatorial structures such as regular triangulations and secondary polytopes, it has been unclear that how such structures are translated in the language of twisted cycles. In this talk, we relate the combinatorial structure of GKZ system to the explicit construction of twisted cycles. It turns out that the intersection form of the twisted homology groups is naturally block-diagonalized with respect to such a basis of cycles. When the regular triangulation is unimodular, we can obtain a closed formula for the intersection numbers. Time permitting, we give a general twisted period relation formula for unimodular triangulations which gives rise to some new quadratic relations for hypergeometric series in several variables.

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Sławomir Michalik
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Gevrey estimate and summability for some moment partial differential equations

In this talk we consider the Cauchy problem for some linear moment partial differential equations.

We describe the Gevrey order and summability of formal solutions of such equations in terms of the Newton polygon.

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Mark Philip F. Ona (a joint work with Jose Ernie C. Lope)
University of Philippines, Diliman, PHILIPPINES

A Nagumo-type theorem on a class of singular first order equations

In 2012, Bacani and Tahara studied the singular first order nonlinear differential equation of the form $t\partial_t u = F(t, x, u, \partial_x u)$, where $F(t, x, u, v)$ is continuous in t and holomorphic in (x, u, v). They proved that a unique solution exists provided that $a(t, x) := \partial_u F(t, x, 0, 0)$ is of order $O(\mu(t))$ and $b(t, x) := \partial_v F(t, x, 0, 0) - \partial_v F(0, x, 0, 0)$ is of order $O(\mu(t))$, as t tends to 0, for some weight function $\mu(t)$.

We revisit this singular equation and present a unique solvability result under weaker assumptions.

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Jiro Sekiguchi (a joint work with M. Kato and T. Mano.)
Tokyo University of Agriculture and Technology, JAPAN

Solutions to extended WDVV equations and Painlevé VI equation

There is a deep relationship between WDVV equation and Painlevé VI equation. We treat this picture to the case of a generalization of WDVV equation called "extended WDVV equation". The purpose of this talk is to report recent progress on the construction of solutions to extended WDVV equation by using 45 algebraic solutions" to Painlevé VI equation.

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Ewa Stróżyna
Warsaw University of Technology, Warszawa, POLAND

Normal forms for vector fields with quadratic leading part

We solve completely the problem of formal classification of vector fields with quadratic leading part using formal changes of coordinates (without changes of time). In the proofs we are able to avoid complicated calculations. The idea of the method is presented in one exemplary case - the vector field with rational first integral and polynomial inverse integrating factor for homogeneous quadratic part of the field.

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Maria Suwińska (a joint work with S. Michalik)
Cardinal Stefan Wyszyński University, Warszawa, POLAND

Hyperasymptotic solution for the heat equation

The main topic of my presentation will be the construction of the hyperasymptotic expansion for the formal solution of the heat equation, when both variables and the initial data function meet a certain specific set of conditions. In particular, I will focus on finding such an expansion away from the Stokes lines of the solution. Moreover, I shall show that the remainders received for every level of hyperasymptotic expansion are exponentially small.

This construction is one of the results of a joint work with S. Michalik.

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Hidetoshi Tahara (a joint work with A. Lastra)
Sophia University, Tokyo, JAPAN

Maillet type theorem for nonlinear totally characteristic PDEs

Let us consider nonlinear singular partial differential equations

$$\text{(E)} \qquad (t\partial_t)^m u = F\big(t, x, \{(t\partial_t)^j \partial_x^\alpha u\}_{j+|\alpha|\le m, j<m}\big)$$

in the complex domain under the assumption that $F(t,x,z)$ (with $z = \{z_{j,\alpha}\}_{j+|\alpha|\le m, j<m}$) is a holomorphic function satisfying $F(0,x,0) \equiv 0$. This equation is classified into the following three types:

Type 1. $(\partial F/\partial z_{j,\alpha})(0,x,0) \equiv 0$ for all (j,α) with $|\alpha| > 0$
Type 2. $(\partial F/\partial z_{j,\alpha})(0,0,0) \neq 0$ for some (j,α) with $|\alpha| > 0$.
Type 3. the other case.

Usually, the equation of type 3 is called a nonlinear totally characteristic type partial differential equation. In this talk, We mainly treat equations of type 3.

We define a non-resonance condition (N), Newton polygon of the equation at $x = 0$, and the generalized Poincaré condition (GP). Then, by using the Newton Polygon, the notion of the irregularity at $x = 0$ of the equation is defined. In the case where the irregularity is greater than one, it is proved under (N) and (GP) that (E) has a unique formal power series solution and it belongs to a suitable formal Gevrey class. The precise bound of the order of the formal Gevrey class is given, and the optimality of this bound is also proved in a generic case.

The result gives a lot of examples of equations which has such a formal power series solution that it is divergent but we don't know whether it is summable or not.

This is a continuation of my talk in FASdiff17 (Alcala, 2017), and is based on a joint work with A. Lastra (University of Alcala).

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Yoshitsugu Takei
Department of Mathematical Sciences, Doshisha University, JAPAN

On the instanton-type expansions for Painlevé transcendents and elliptic functions

As was verified by Kapaev and others, the most basic Stokes phenomena for Painlevé transcendents are described by formal power series solutions and transseries solutions of Painlevé equations. However, to discuss more general Stokes phenomena, we need to deal with instanton-type solutions, which are purely formal and whose behavior are much more wild than transseries solutions. In this talk, from the viewpoint of exact WKB analysis, we investigate instanton-type formal solutions of Painlevé equations and those of the equation for Weierstrass' elliptic functions. After explaining the construction of instanton-type solutions, we discuss how to give an analytic meaning to them. In particular, in the case of the equation for Weierstrass' elliptic functions it turns out that instanton-type solutions are nothing but Fourier expansions of elliptic functions.

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Yumiko Takei (a collaboration with K. Iwaki and T. Koike ([3]))
Kobe University, JAPAN

Voros coefficients for hypergeometric differential equations and Eynard-Orantin's topological recursion

Exact WKB analysis is a powerful tool to study global behavior of solutions of differential equations and Voros coefficients play an important role in performing such a global study through exact WKB analysis. On the other hand, the topological recursion is introduced by B. Eynard and N. Orantin ([2]) to study the correlation functions in the random matrix theory and it gives a gen- eralization of the loop equations for the matrix model. Recently, a surprising connection between exact WKB analysis and topological re- cursion has been discovered, that is, it is discovered that WKB solutions are constructed via the topological recursion ([1]). In this talk, we prove that the Voros coefficients for hypergeometric differential equations are described by the generating functions of free energies defined in terms of the topological recursion. Furthermore, as its applications we show the following objects can be computed in an explicit manner: (i) three-term difference equations that the generating function of the free energies satisfies, (ii) concrete form of the free energies, and (iii) Voros coefficients for hypergeometric equations.

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[3] Iwaki, K. , Koike, T. and Takei, Y., Voros coefficients for the hypergeometric differential equations and Eynard-Orantin's topological recursion, arXiv:1805.10945.

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Kouichi Takemura
Chuo University, JAPAN

On q-deformations of the Heun equation

Heun's differential equation is a standard form of the second order linear differential equation with four regular singularities on the Riemann sphere, and it is written as

$$\frac{d^2y}{dz^2} + \left(\frac{\gamma}{z} + \frac{\delta}{z-1} + \frac{\epsilon}{z-t}\right)\frac{dy}{dz} + \frac{\alpha\beta z - B}{z(z-1)(z-t)}y = 0,$$

with the condition $\gamma + \delta + \epsilon = \alpha + \beta + 1$.

In [2], q-difference deformations of Heun's differential equation were obtained by degenerations of Ruijsenaars-van Diejen operators, which were also obtained in connection with q-Painlevé equations. One of the q-deformation is the q-Heun equation written as

$$(9)\quad \{a_0 + a_1x + a_2x^2\}g(x/q) + \{b_0 + b_1x + b_2x^2\}g(x) + \{c_0 + c_1x + c_2x^2\}g(qx) = 0.$$

where $a_0a_2c_0c_2 \neq 0$. The equation (9) was discovered by Hahn [1] in 1971.

The q-Heun equation was obtained in [2] as an eigenfunction of the fourth degeneration of the Ruijsenaars-van Diejen operator of one variable

$$A^{\langle 4\rangle} = x^{-1}(x - q^{h_1+1/2}t_1)(x - q^{h_2+1/2}t_2)T_{q^{-1}} + q^{\alpha_1+\alpha_2}x^{-1}(x - q^{l_1-1/2}t_1)(x - q^{l_2-1/2}t_2)T_q$$
$$- \{(q^{\alpha_1} + q^{\alpha_2})x + q^{(h_1+h_2+l_1+l_2+\alpha_1+\alpha_2)/2}(q^{\beta/2} + q^{-\beta/2})t_1t_2x^{-1}\}$$

with eigenvalue E, where $T_{q^{-1}}g(x) = g(x/q)$ and $T_qg(x) = g(qx)$. Namely the equation (9) is written as $A^{\langle 4\rangle}g(x) = Eg(x)$.

We can obtain variants of the q-Heun equation as eigenfunctions of the third or second degeneration of the Ruijsenaars-van Diejen operator of one variable. Then we investigate local properties of the q-Heun equation and its variants. In particular we characterize the variants of the q-Heun equation by using analysis of regular singularities [3]. We also consider the quasi-exact solvability of the q-Heun equation and its variants. Namely we investigate finite-dimensional subspaces which are invariant under the action of the q-Heun operator or variants of the q-Heun operator [3].

References

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Mika Tanda (a joint work with T. Aoki and T. Takahashi)
Kwansei Gakuin University, JAPAN

The asymptotic expansions of the hypergeometric function with respect to a parameter

We consider the Gauss hypergeometric differential equation with a large parameter from the viewpoint of the exact WKB analysis. We introduce a large parameter η in the parameters of the Gauss hypergeometric equation as general linear forms of η. As is well known, the Gauss hypergeometric differential equation has a system of fundamental solutions in the neighborhood of the origin (u_1, u_5) which are expressed in term of hypergeometric functions. On the other hand, the Gauss hypergeometric differential equation with a large parameter has another system of fundamental solutions which are defined by Borel sums (Ψ_+, Ψ_-) of WKB solutions. We investigate linear relations which hold between (u_1, u_5) and (Ψ_+, Ψ_-) . By using these relations, we give asymptotic expansion formulas for the hypergeometric function with respect to the large parameter.

This is a collaboration with Takahi Aoki and Toshinori Takahashi.

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Bożena Tkacz (a joint work with S. Michalik)
Cardinal Stefan Wyszyński University, POLAND

The Stokes phenomenon for some moment partial differential equations

We consider the Stokes phenomenon for the solutions of general homogeneous linear moment partial differential equations with constant coefficients in two complex variables under condition that the Cauchy data are holomorphic on the complex plane but finitely many singular or branching points with the appropriate growth condition at the infinity. The main tools that we use are the theory of summability and multisummability, and the theory of hyperfunctions.

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Paweł Wójcicki
Warsaw University of Technology, POLAND

The weighted Bergman kernel and the Green's function

In this talk we present the results from the paper [1] together with some additional remarks on the connection between the weighted Bergman kernel and the unweighted one.

References

[1] Krantz, S., Wójcicki, P. M. *The weighted Bergman kernel and the Green's function*, Complex Anal. Oper. Theory 11 (2017), no. 1, 217–225.

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Hiroshi Yamazawa
Shibaura Institute of Technology, Saitama, JAPAN

Gevrey and q-Gevrey asymptotics for some linear q-difference differential equations

Let $(t,x) \in \mathbb{C}^2$. We study the following equation:

$$u + t\sigma_q^s u + t\sigma_q^s \partial_x u = f(t,x) + t\sigma_q^{s'} \partial_x^2 u \tag{E}$$

for $0 < s' < s$ where $f(t,x)$ is a holomorphic function in a neighborhood of the origin with $f(0,x) \equiv 0$.

In this talk we will introduce results of Gevrey and q-Gevrey asymptotic expansion of solutions for the equation (E).

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Masafumi Yoshino
Hiroshima University, JAPAN

The linearization problem for holomorphic vector fields and parametric Borel summability

We consider the linearization of a holomorphic vector field in some neighborhood of the origin of $\mathbb{C}^n$. This is a classical problem and there are many important works since Poincaré's. As it is well known, we often encounter the divergence due to the small denominator problem. Existence of the linearizing transformation was proved under Diophantine condition and nonresonance condition (Siegel, Stolovitch). Diophantine condition is useful in order to control the divergence, while there is another way to controle divergence by considering first integral (Ito, Zung).

In view of these works, we shall study how to understand the divergence of the linearizing transformation in terms of Borel summability theory. More precisely, we consider the so-called homology equation which the linearizing transformation satisfies. Instead of solving the equation in a usual way, we first construct a solution by the formal power series expansion of some parameter in the equation. Then we construct a linearizing transformation in terms of the parametric Borel summability. Finally, we make the analytic continuation of the Borel sum to obtain the Poincaré's classical theorem. Some extension of the method to the small denominator problem is also presented.

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[1] Yoshino, M., Analytic continuation of Borel sum of formal solution of semilinear partial differential equation, *Asymptotic Analysis*, **92**, (2015), 65-84. DOI 10.3233/ASY-141270.

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Michał Zakrzewski
Jan Kochanowski University in Kielce, POLAND

Relations between multiple zeta values arising from geometry of symmetric spaces

Multiple zeta values are defined by the series

$$\text{(sMZV)} \qquad \zeta(s_1, ..., s_r) := \sum_{n_1,...,n_k>0} l_1^{-s_1} ... l_r^{-s_r},$$

where l_i are affine forms in k variables $n_j \in \mathbb{N}$, with positive integral coefficients and s_i are natural numbers, such that the series (sMZV) converges.

To multiple zeta values one can associate certain (generalized) hypergeometric functions. For example, the Riemann zeta value $\pi^2/6 = \zeta(2)$ is associated to ${}_3F_2(1,1,1;2,2\,|\,t)$. Such hypergeometric functions can be understood in terms of sections of certain line bundle over Graßmannian manifold $G_{k,n}$. In our talk we will present some relations between multiple zeta values arising from geometric properties of Graßmannians.

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Federico Zullo
DICATAM, Università di Brescia, ITALY

On the solutions of the Airy equation

We present some observations on the distribution of zeros of solutions of the Airy equation in the complex plane. A recursion for the zeros and a characterization of their distribution through a sequence of polynomials will be given. A representation of the solutions in terms of certain sequences involving integers will be discussed. The applicability of the results beyond the Airy equation will be considered.

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Henryk Żołądek
Warsaw University, POLAND

Invariants of group representations, dimension/degree duality and normal forms of vector fields

We develop a constructive approach to the problem of polynomial first integrals for linear vector fields. As an application we obtain a new proof of the theorem of Wietzenböck about finiteness of the number of generators of the ring of constants of a linear derivation in a polynomial ring. Moreover, we give an alternative proof of the analyticity of the normal form reduction of a germ of vector field with nilpotent linear part in a case considered by Stolovich and Verstringe.

The Banach Center School and Conference

Complex Differential and Difference Equations

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List of participates with their email addresses and titles of talks

Tanuja Adaviswamy (Siddaganga Institute of Technology), a.tanuja1@gmail.com
Meromorphic solution of Ginzburg-Landau family of equations with variable coefficients using Nevanlinna theory
Yulia Bibilo (Concordia University), y.bibilo@gmail.com
Application of the Painlevé 3 equation to the Josephson effect
Ewa Ciechanowicz (University of Szczecin), ewa.ciechanowicz@usz.edu.pl
Value distribution and growth of solutions of certain nonlinear ODEs
Jan Dereziński (Warsaw University), jan.derezinski@fuw.edu.pl
From the conformal group to symmetries of hypergeometric type equations I–V
Galina Filipuk (Warsaw University), Filipuk@mimuw.edu.pl
Orthogonal polynomials on non-uniform lattices
Pilar R. Gordoa (Universidad Rey Juan Carlos), pilar.gordoa@urjc.es
Matrix Painlevé hierarchies
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The Dirichlet type problem for complex polyharmonic functions
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Non-generic isomonodromy deformations
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Asymptotic analysis for a confluent KZ equation in two variables
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A necessary condition for k-summability of formal solutions to some linear q-difference-differential equations
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Dimensioned numbers and differential equations
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Resurgent structures in differential equations I–V,
Multisummability and strongly regular sequences
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Complex differential equations with movable algebraic singularities
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The Riemann-Hilbert problem and singularity formation in the localized induction approximation
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Mean values and heat type equations
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On parametric Borel summability for linear singularly perturbed Cauchy problems with linear fractional transforms
Toshiyuki Mano (University of the Ryukyus), tmano@math.u-ryukyu.ac.jp
Analytic representation of potential vector fields and isomonodromic τ functions
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Intersection theory for Euler integral representations of GKZ hypergeometric functions

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Gevrey estimate and summability for some moment partial differential equations
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Different aspects of summability I–V
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A Nagumo-type theorem on a class of singular first order equations
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Solutions to extended WDVV equations and Painlevé VI equation
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Algebro-geometric method for algebraic differential equations II, IIIb, IV
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Gevrey and q-Gevrey asymptotics for some linear q-difference differential equations
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Movable singularity of some Hamiltonian system and blowup of semilinear wave equation I–V,
The linearization problem for holomorphic vector fields and parametric Borel summability

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