

For every regular convex cone $K \subset \mathbb{R}^3$ there exists a unique complete hyperbolic affine 2-sphere with mean curvature -1 which is asymptotic to the boundary of the cone. Two cones are associated with each other if the Blaschke metrics of the corresponding affine spheres are related by an orientation-preserving isometry. We call a cone self-associated if it is linearly isomorphic to all its associated cones. We give a complete classification of the self-associated cones and compute isothermal parametrizations of the corresponding affine spheres. The solutions can be expressed in terms of degenerate Painlevé III transcendents. The boundaries of generic self-associated cones can be represented as conic hulls of vector-valued solutions of a certain third-order linear ordinary differential equation with periodic coefficients. The technique developed in this paper can also be applied to the three-dimensional semi-homogeneous cones, with similar results.