Lagrangian submanifolds of the complex quadric

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Many of the particularly interesting surface classes are characterized by the harmonicity of some "Gauss map". This includes a general construction procedure using loop groups and a Sym formula.The complex quadric Q^n is the complex hypersurface of complex $(n+1)$-dimensional projective space given in homogeneous coordinates by the equation $z_0^2+z_1^2+\ldots+z_{n+1}^2=0$. This manifold inherits a Kähler structure from the complex projective space, carries a family of non-integrable almost product structures and its curvature can be relatively easily described in terms of these two. Moreover, Q^n is the natural target space when considering the Gauss map of a hypersurface of a round sphere. In fact, such Gauss maps are related to minimal Lagrangian submanifolds of Q^n. We will discuss this relation – in particular for isoparametric hypersurfaces of spheres – and then study minimal Lagrangian submanifolds of Q^n, obtaining examples and some classifications, such as that of minimal Lagrangian submanifolds of Q^n with constant sectional curvature.