SOME APPLICATIONS OF THE AFFINE QUASI-EINSTEIN EQUATION

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(joint work with P. Gilkey)

An affine manifold (M, ∇) is said to be *affine quasi-Einstein* if there is a solution $f \in \mathcal{C}^\infty(M)$ of the equation

$$\mathrm{Hes}_f^\nabla - \mu f \rho_{\mathrm{sym}}^\nabla = 0 \tag{1}$$

for some $\mu \in \mathbb{R}$, where Hes_f^∇ and $\rho_{\mathrm{sym}}^\nabla$ denote the Hessian of f and the symmetric Ricci tensor, respectively. Solutions to this equation provide self-dual quasi-Einstein metrics of signature $(--++)$ which are not necessarily locally conformally flat by means of the Riemannian extension of the affine connection ∇ to T^*M.

The dimension of the space of solutions of the affine quasi-Einstein equation corresponding to $\mu = -1$ provides a strong projective invariant for affine surfaces. In particular, strongly projectively flat connections can be (locally) parametrized by solutions of the affine quasi-Einstein equation [1]. Our purpose in this lecture is to show how this distinguished parametrization may be used to investigate different geometric properties: linear equivalence [2] and geodesic completeness of (M, ∇).

References

[1] M. Brozos-Vázquez, E. García-Río, P. Gilkey, and X. Valle-Regueiro, "A natural linear equation in affine geometry: the affine quasi-Einstein equation", Proc. Amer. Math. Soc. **146** (2018), no. 8, 3485–3497.

[2] P. Gilkey, and X. Valle-Regueiro, "Applications of PDEs to the study of affine surface geometry", arXiv:1806.06789.