On submanifolds of 3-sphere immersions

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Dedicated to Professor Udo Simon's 80th birthday

In this lecture, I will talk about some researches on submanifolds of the 3-sphere immersions. This includes in particular our recent study with ambient spaces the complex projective space and the homogeneous nearly Kähler manifold $S^3 \times S^3$.

For the former case, we notice that the equivariant CR minimal immersions from the round 3-sphere S^3 into the complex projective space $\mathbb{C}P^n$ have been classified by Zhenqi Li explicitly (J London Math Soc **68** (2003), 223-240). Then, by employing the equivariant condition which implies that the induced metric is left-invariant, and that all geometric properties of $S^3 = \mathrm{SU}(2)$ endowed with a left-invariant metric can be expressed in terms of the structure constants of the Lie algebra $\mathfrak{su}(2)$, we establish an extended classification theorem for equivariant CR minimal immersions from the 3-sphere S^3 into $\mathbb{C}P^n$ without the assumption of constant sectional curvatures.

For the latter case, we first show that isotropic Lagrangian submanifolds in a 6-dimensional strict nearly Kähler manifold are totally geodesic. Then, under some weaker conditions, a complete classification of the J-isotropic Lagrangian submanifolds in the homogeneous nearly Kähler $S^3 \times S^3$ is obtained, which mainly consists of 3-sphere immersions. Here, a Lagrangian submanifold of $S^3 \times S^3$ is called J-isotropic, if there exists a function λ, such that the metric g, the almost complex structure J and the second fundamental form h satisfy $g((\nabla h)(v, v, v), Jv) = \lambda$, for all unit tangent vector v.

References

[1] Zejun Hu, Jiabin Yin and Zhenqi Li, *Equivariant CR minimal immersions from S^3 into $\mathbb{C}P^n$*, Ann. Global Anal. Geom., **54** (2018), 1-24.

[2] Zejun Hu and Yinshan Zhang, *On isotropic Lagrangian submanifolds in the homogeneous nearly Kähler $S^3 \times S^3$*, Sci. China Math., **60** (2017), 671-684.