Book of abstracts

Homotopy algebras, deformation theory and quantization

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I. Minicourses

Giovanni Felder ETH Zürich

Derived representation schemes and supersymmetric gauge theory

Representation schemes parametrize finite dimensional representations of given associative algebras. They can be very singular objects and their derived version is better behaved. We will provide an introduction to this subject, based on examples, and sketch the (mostly conjectural) relation with equivariant K-theory of Nakajima varieties and supersymmetric gauge theory.

Simone Gutt Universite Libre de Bruxelles

Deformation theory and group actions

This will be a short introduction to deformation quantization as a general method to pass from a classical mechanical system (represented by a Poisson manifold) to the corresponding quantum system. The emphasis in this approach is put on the algebra of observables (functions on the Poisson manifold) , quantization appearing as a deformation of the algebra structure.

Given a classical symmetry, given by the action of a Lie group or of a Lie algebra on the manifold, preserving the Poisson structure, one is interested in building a quantization such that the Lie group acts by automorphisms of it (or the Lie algebra by derivations). A classical action is particularly interesting when it comes from a momentum map and one looks in this case for a corresponding momentum map in the corresponding quantum setting. A classical construction using a momentum map is the Marsden-Weinstein reduction which allows to reduce the number of variables and one looks for quantum analogues of reduction in the quantum framework.

The course will concentrate on formal deformation quantization, stating results concerning those points. Some comments will be made about the necessity for a convergent setting, and the use of group actions to build examples in that setting.

Sergei Merkulov University of Luxembourg

Graph complexes in algebra and geometry - recent advances

Graph complexes have been introduced by Maxim Kontsevich in early 90s as a purely combinatorial tool for computing very important invariants in geometry and algebra (e.g., cohomology groups of moduli spaces of algebraic curves with punctures, universal obstructions to a perturbative quantization of Poisson structures, etc.). His works on graph complexes attracted much attention but, unfortunately, did not lead immediately to really new results – it turned out to be very hard to compute the cohomology of that new "simple" gadgets – the graph complexes.

Recently there was a huge progress in computations of cohomology groups of graph complexes due to Thomas Willwacher and his collaborators. This progress lead us to a complete classification of universal quantizations of Poisson structures and Lie bialgebras, and give us a surprisingly clean picture of how the mysterious Grothendieck-Teichmueller groups acts as symmetries on polyvector fields, on Lie bialgebras, etc.

The main purpose of my two lectures is to give an introduction to the theory to graph complexes. The plan includes: 1) basic definitions; 2) operadic compositions of graphs; 3) the definition of the differential on ordinary and ribbon graph complexes; 4) outline (without proofs) some recent applications of the theory of graph complexes in algebra and geometry.

Ping Xu Penn State University

Dg manifolds, formality theorem and Kontsevich-Shoikhet conjecture

Dg manifolds (or Q-manifolds) are a useful geometric notion which unifies many important structures such as curved L_∞ algebras and derived intersections.

We will give an introduction to dg manifolds and its relation with derived differential geometry.

The Todd class of dg manifolds extends both the classical Todd class of complex manifolds and the Duflo element of Lie theory. We establish a formality theorem for finite-dimensional smooth dg manifolds. As an application, we prove the Kontsevich–Shoikhet conjecture: a Kontsevich–Duflo type theorem holds for all finite-dimensional smooth dg manifolds. The minicourse is based on joint works with Kai Behrend, Hsuan-Yi Liao and Mathieu Stienon.

II. Invited talks

Martin Bordemann Université de Haute Alsace

(Bi)modules in smooth deformation quantization: old and new, and (non)formality

We shall deal with the simultaneous deformation of associative algebras and their (bi)modules applied to deformation quantization on smooth manifolds. There is an obvious formulation in terms of differential graded Lie algebras already mentioned by D. Arnal et al in 1983. We discuss two particular cases: 1. the space of smooth functions on a submanifold (which has to be coisotropic due to Gabber's Theorem), 2. the space of of smooth fonctions on the total space of a fibered manifold over a Poisson manifold. In both cases we compute the corresponding cohomologies (in terms of differential Hochschild cohomology), related this to the differential geometry (e.g. adapted poly-vectorfields), and check L_∞ formality. This is joint work with Benedikt HURLE, Mulhouse.

Kenji Iohara Institut Camille Jordan, Université Lyon 1

On Lie algebras of generalized Jacobi matrices

The Lie algebra in question is the one studied in relation with the soliton theory, in particular, the KP Hierarchy. Here, we discuss about its homology and its possible applications. This talk is based on collaborations with A. Fialowski.

Andrey Lazarev Lancaster University

Strong homotopy and differential graded categories associated to topological spaces and simplicial sets

(joint with J. Chuang and J. Holstein)

The modern formulation of formal deformation theory in characteristic zero is based on the notion of a Maurer-Cartan (MC) element in a differential graded (dg) Lie algebra L; it is an odd element x satisfying the flat connection equation $dx + 1/2[x, x] = 0$. There is a suitable notion of gauge equivalence and the moduli set of MC elements in dg Lie algebras. There is an analogous but somewhat less well studied notion of an MC element in a dg associative algebra A and the corresponding MC moduli set. The latter is not a quasi-isomorphism invariant of A, but is an invariant under a finer strong homotopy equivalence relation on dg algebras. I will explain this notion, and how it gives rise to various homotopy invariants and Riemann-Hilbert type theorems. If time permits, I will outline connections with Joyals quasi-categories.

Michael Penkava University of Wisconsin-Eau Claire

Infinity and cyclic infinity algebras, cohomology and applications

Infinity algebras, also known as strongly homotopy algebras, were introduced by Stasheff in the context of associahedra, as an algebraic object connected to topology. He also realized that the cohomology of both Lie and Associative algebras could be given in terms of an intrinsic bracket on a space of coderivations of an appropriate coalgebra. It has been established that versal deformations for both the L_∞ and A_∞ algebras exist and can be constructively obtained, under mild assumptions on the cohomology. Cyclic cohomology is related to the deformations of the algebra structure preserving an invariant inner product, and cyclic algebras give rise to homology cycles on appropriate graph complexes. The celebrated solution to the existence problem for deformation quantization of Poisson manifolds by Kontsevich uses an L_∞ isomorphism as a key tool. We explore various infinity algebras and cyclic versions.

Norbert Poncin University of Luxembourg

Derived D-geometry

Derived $\mathcal{D}$-geometry is a combination of derived (or homotopical) algebraic geometry and algebraic $\mathcal{D}$-geometry. Homotopical algebraic geometry is algebraic geometry considered from the viewpoint of homotopical mathematics ('equalities replaced by homotopies'). Algebraic $\mathcal{D}$-geometry is the geometry of $\mathcal{D}_X$-schemes, i.e., of X-schemes equipped with an integrable connection along a smooth scheme X. More specifically, the opposite of the category of affine $\mathcal{D}_X$-schemes is the category of commutative unital algebras over the sheaf $\mathcal{D}_X$ of differential operators of X. One of the interesting concepts of derived $\mathcal{D}$-geometry is the notion of derived $\mathcal{D}_X$-stack, which we will explain using the functor of points approach. This type of stack appears in particular when dealing with bad intersections or quotients by badly behaved actions. Its most important aspect is the encoded action by ('total or horizontal') differential operators. The purpose of the talk is to provide evidence for derived $\mathcal{D}$-geometry being the convenient setting for a coordinate-free study of the moduli space of solutions of a system of PDEs modulo symmetries, in particular for the Batalin-Vilkovisky complex in gauge theories.

Christoph Schweigert University of Hamburg

More about modular functors

We show that the modular group $SL(2,\mathbb{Z})$ acts the Hochschild complex of a finite-dimensional factorizable ribbon Hopf algebra. We construct similar complexes for more general mapping class groups. Our results can be seen as a first step towards a derived modular functor.

Oleg Sheinmann Steklov Mathematical Institute

Quantization of Lax integrable systems

We consider a general formalism for a wide range of finite-dimensional integrable systems possesing a Lax representation, and give a general procedure of prequantization of such systems. Our procedure can also be interpreted as a correspondence between Lax integrable systems and Conformal Field Theories. To be more specific, we will construct a unitary projective representation of the Lie algebra of Hamiltonian vector fields of the system by means of covariant derivatives by virtue of a kind of high genus Knizhnik-Zamolodchikov connection. For example, in the case of Hitchin systems our approach is an alternative of the quantization by means of the Hitchin connection in the same sense as Knizhnik-Zamolodchikov connection is an alternative for the Hitchin connection. The deformation theme is involved in the subject via "deformation of Tyurin parameters" providing missing parameters for the Lax equations with the spectral parameter on a Riemann surface. In the case of Hitchin systems, contraction of this deformation gives interesting integrable systems of algebraic origin

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III. Contributed talks

Ziemowit Domański Poznań University of Technology

Deformation quantization on the cotangent bundle of a Lie group

We present a complete theory of non-formal deformation quantization on the cotangent bundle of a Lie group. An appropriate integral formula for the starproduct is introduced together with a suitable space of functions on which the star-product is well defined. This space of functions becomes a Fréchet algebra as well as a pre-C^*-algebra. Basic properties of the star-product are proved and the extension of the star-product to a Hilbert algebra and an algebra of distributions is given. A C^*-algebra of observables and a space of states are constructed. Moreover, an operator representation in position space is presented.

Arthemy Kiselev Johann Bernoulli Institute, University of Groningen

The Kontsevich graph orientation morphism revisited

The orientation morphism $\vec{Or} : \gamma \to \dot{\mathcal{P}}$ associates differential-polynomial flows $\dot{\mathcal{P}} = \mathcal{Q}(\mathcal{P})$ on spaces of bi-vectors $\mathcal{P}$ on finite-dimensional affine manifolds N^n with (sums of) finite unoriented graphs Γ with ordered sets of edges and without multiple edges and one-cycles. It is known that d-cocycles γ with respect to the vertex-expanding differential d are mapped by $\vec{Or}$ to Poisson cocycles $\mathcal{Q}(\mathcal{P}) \in \ker[[\mathcal{P}, \cdot]]$, that is, to infinitesimal symmetries of Poisson bi-vectors $\mathcal{P}$. The formula of orientation morphism $\vec{Or}$ was expressed by Kontsevich (1996), see also C. Jost (2013), in terms of the edge orderings as well as parity-odd and parity-even derivations on the odd cotangent bundle $\Pi T^* N^n$. By inspecting the rules of signs which are suggested by that algebraic formula of $\vec{Or}$, we summarise its defining properties in terms of oriented graphs with edge orderings at all internal vertices. The reasoning is illustrated by the tetrahedral and by pentagon and heptagon-wheel cocycle flows.

Hany Nasry Military Technical College, Cairo

Clifford algebra in sensor fusion and coordinate transformation

One of the main issues within sensor fusion, in robotics applications and calibration, is the transformation of coordinates or variables between different reference frames. This transformation is carried out using different approaches; among them are the quaternion parameters and direction cosines. The nature of applications necessitates fast in computation and easy or concise in manipulation. Therefore, the objective of this paper is to use Geometric algebra in coordinate transformation. Using Geometric algebra, we find that this formulation is simpler, with lower computational time and more obvious than other methods as it depends on the characteristics of Geometric algebra which unites vectors of different planes into a single mathematical system with a comprehensive geometric significance.

Andriy Panasyuk University of Warmia and Mazury

On linear-quadratic Poisson pencils on $\mathfrak{gl}(3)$

In a recent paper Vladimir Sokolov introduces a three-parametric family of quadratic Poisson structures on $\mathfrak{gl}(3)$ each of which is compatible with the canonical linear Poisson bracket. The complete involutive family of polynomial functions related to these bi-Poisson structures contains the Hamiltonian of the so-called elliptic Calogero-Moser system, the quantum version of which is also discussed in the same paper.

We show that there exists a 10-parametric family of quadratic Poisson structures on $\mathfrak{gl}(3)$ compatible with the canonical linear Poisson bracket and containing the Sokolov family. The quantization matters will be also touched in this talk.

(The joint work with Ihor Mykytyuk.)

Leonid Ryvkin Ruhr-Universität Bochum

Darboux type theorems in multisymplectic geometry

Multisymplectic geometry was developed to give a finite-dimensional Hamiltonian description of classical field theory, in analogy to symplectic geometry describing classical mechanics. A key tool in symplectic geometry is the existence of local standard coordinates, assured by the Darboux theorm. This classical theorem, stating that in appropriate coordinates all symplectic forms locally take the form $dx^1 \wedge dx^2 + \ldots + dx^{2n-1} \wedge dx^{2n}$, fails to naively generalize to the multisyplectic situation (i.e. to non-degenerate closed forms of arbitrary degree). We will determine the necessary and sufficient conditions for Darboux type theorems for multisymplectic manifolds of product and complex types. Then, we will discuss possible obstructions to the ω-transitivity of the diffeomorphism groups of these multisymplectic structures.

Akifumi Sako Tokyo University of Science

Twisted Fock representation of Kähler manifolds and Hermitian-Einstein metrics from noncommutative $U(1)$ instantons

Joint work with: Kentaro Hara and Hyun Seok Yang.

The Fock representation of noncommutative Kähler manifolds and its applications are discussed. Especially, Hermitian-Einstein metrics are constructed from noncommutative instantons on $\mathbb{C}^2$. Noncommutative Kähler manifolds studied here are constructed by deformation quantization with separation of variables. This deformation quantization was given by Karabegov. The algebra of the noncommutative Kähler manifolds contains the Heisenberg-like algebras. The algebras on noncommutative Kähler manifolds are represented as linear operators acting on the Fock space. Using the Fock representations, physical quantities in noncommutative Kähler manifolds are given by explicit functions. As an example of the application of the Fock representation, we make Hermitian-Einstein metrics. Hermitian-Einstein metrics are locally constructed by using self-dual two forms. As the self-dual two forms, $U(1)$ instantons on a noncommutative $\mathbb{C}^2$ are used here. To construct the noncommutative instantons on $\mathbb{C}^2$ we use the Fock representation. There is a dictionary between the basis of the Fock representations and ordinary functions on the Kähler manifolds. Using the dictionary concrete examples of Hermitian-Einstein metrics are obtained. This correspondence between the Hermitian-Einstein metrics and the noncommutative $U(1)$ instantons is deeply related to the Seiberg-Witten map. Kähler conditions are concerned with the Bianchi identities for $U(1)$ gauge curvatures in commutative space.

Yunhe Sheng Jilin Universit

Deformations and their controlling cohomologies of $\mathcal{O}$-operators (generalized Rota-Baxter operators)

$\mathcal{O}$-operators are important in broad areas in mathematics and physics, such as integrable systems, the classical Yang-Baxter equation, pre-Lie algebras and splitting of operads. In this paper, we establish a deformation theory of $\mathcal{O}$-operators which is consistent with the general principles of deformation theories. On the one hand, we show that $\mathcal{O}$-operators are characterized as the Maurer-Cartan elements in a suitable graded Lie algebra. We also construct a differential graded Lie algebra from an $\mathcal{O}$-operator and show that deformations of the $\mathcal{O}$-operator are characterized as Maurer-Cartan elements in this differential graded Lie algebra. On the other hand, we identify a Lie algebra with a representation induced from an $\mathcal{O}$-operator T such that the corresponding Chevalley-Eilenberg cohomology controls deformations of T, thus can be regarded as an analogue of the Andrè-Quillen cohomology for the $\mathcal{O}$-operator. Thereafter, we study infinitesimal and formal deformations of $\mathcal{O}$-operators. In particular, we introduce the notion of Nijenhuis elements to characterize trivial infinitesimal deformations. Formal deformations and extendibility of order n deformations of an $\mathcal{O}$-operator are also characterized in terms of the new cohomology theory. Applications are given to deformations of Rota-Baxter operators and skew-symmetric r-matrices for the classical Yang-Baxter equation. For skew-symmetric r-matrices, there is an independent Maurer-Cartan characterization of the deformations as well as an analogue of the Andrè-Quillen cohomology controlling deformations, which turn out to be equivalent to the ones obtained as $\mathcal{O}$-operators associated to the coadjoint representation. Finally, we study infinitesimal deformations of skew-symmetric r-matrices and the corresponding triangular Lie bialgebras.

IV. Young Researchers Session

Jill Ecker University of Luxembourg

The Low-Dimensional Algebraic Cohomology of the Virasoro Algebra

The main focus of this presentation lies on the proof of the one-dimensionality of the third algebraic cohomology of the Virasoro algebra with values in the adjoint module. Because we are working with pure algebraic cohomology, our results are valid for any concrete realization of the Witt and the Virasoro algebra.

The talk starts with a brief introduction of the Witt and the Virasoro algebra. In a second step, the Chevalley-Eilenberg cohomology of Lie algebras is described, including the description of tools for computing this cohomology, such as the Hochschild-Serre spectral sequence. The proof of our main result consists of two parts. The first part consists in proving that the third algebraic cohomology of the Virasoro algebra with values in the Witt algebra is isomorphic to the third cohomology of the Witt algebra with values in the adjoint module. This proof uses the Hochschild-Serre spectral sequence. The second part consists in proving the one-dimensionality of the third algebraic cohomology of the Witt and the Virasoro algebra with values in the trivial module. Although the second part uses elementary algebra, the proof per se is not elementary, but somewhat intricate. This is joint work with Martin Schlichenmaier.

The vanishing of the third algebraic cohomology of the Witt algebra with values in the adjoint module, as well as the vanishing of the first algebraic cohomology of the Witt and the Virasoro algebra, have already been proven by Ecker and Schlichenmaier. The vanishing of the second algebraic cohomology of the Witt and the Virasoro algebra was shown by Schlichenmaier, see also Fialowski.

Satyendra Kumar Mishra Indian Institute of Technology

On Deformations of Courant pairs and Poisson algebras

This work provides a deformation theory for a new type of algebraic structure called Courant pair – a special type of Courant algebra over the Lie algebra of derivations of an associative algebra. One can find that Courant algebras appear in several branches of mathematics. One of the main problems in deformation theory is to describe all non-equivalent deformations of a given object. We solve this problem by defining the required deformation cohomology and systematically develop a method of inductive construction giving a versal deformation of Courant pairs. Since every Leibniz pair is an example of a Courant pair, it is natural to ask for additional deformations for a Poisson algebra or a Leibniz pair by considering it in the larger category of Courant pairs. We answer this question by computing some Poisson algebra examples and find that there is a difference even in the infinitesimal level. We explicitly compute universal infinitesimal deformations of Poisson algebra structures on the three dimensional complex Heisenberg Lie algebra by considering them in both the categories of Leibniz pairs and Courant pairs. More interestingly, a Courant pair can be described as a non skew-symmetric version of Open-closed homotopy algebra and subsequently as an algebra over an operad.

Safdar Quddus Indian Institute of Science

Noncommutatative algebraic torus and its algebraic properties

In few articles by Yan Soibelman, the non-commutative algebraic tori were viewed as the points of compactification of the moduli space of elliptic curves. This is regarded as one of the basic model in the non-commutative algebraic geometry. Several algebraic geometric properties of the same have been studied in using the mechanisms of Functional analysis and Category theory. In this talk I plan to study some of the cohomological properties of the same under the action of discrete subgroups of $SL(2, \mathbb{Z})$. We also explore the Chern–Connes indices and the talk about the K-theory.

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Marcel Rubió KU Leuven

Homotopy of singular varieties via L_∞ pairs

In this talk we show that for a complex algebraic variety with no weight-zero 1-cohomology classes, the fundamental group is strongly restricted; in particular, the irreducible components of the cohomology jump loci of rank one local systems containing the constant sheaf are complex affine tori. We prove this by studying the cohomology jump loci (or generalized Brill-Noether loci) via L_∞ pairs: the yoga here being that a deformation problem with cohomology constraints is governed by an L_∞ pair (L, M), consisting of an L_∞ algebra L and an L-module M. The results we obtain are in contrast to the work by Simpson, Kapovich and Kollár, stating that every finitely presented group is the fundamental group of an irreducible complex algebraic variety with only normal crossings and Whitney umbrellas as singularities. This is joint work with Nero Budur.

Kirill Salmagambetov Higher School of Economics

Towards derived algebraic supergeometry

Supergeometry in the usual sense studies manifolds whose structure sheaf is a $\mathbb{Z}_2$-graded algebra. Sometimes one also would like to construct a manifold whose structure sheaf is a $\mathbb{Z}_2$-graded algebra together with an odd derivation of square zero (this is called homological vector field). For example, one can ask: what kind of geometric object corresponds to a differential $\mathbb{Z}_2$-graded algebra?

In my talk I will explain that this question is in general too complicated. However, one can introduce a certain class of filtered differential $\mathbb{Z}_2$-graded algebras which can be studied by means of derived algebraic geometry. If time permits, I will outline some applications to deformation theory in the $\mathbb{Z}_2$-graded case and supermoduli spaces arising in mathematical physics.

V. Poster session

Matt Booth University of Edinburgh

The derived contraction algebra

Derived categories have a strong link to birational geometry – for example, Bridgeland's famous result that threefold flops induce derived equivalences. Wemyss's Homological Minimal Model Programme is an attempt to run the MMP using derived methods. In particular, given a threefold flopping contraction, one can associate a certain finite-dimensional algebra as an invariant, called the contraction algebra. It controls the noncommutative deformation theory of the flopping curves, and is conjectured to determine completely the complete local geometry of the base. In this poster, I'll describe a natural – at least from the point of view of noncommutative derived geometry –

generalisation, the derived contraction algebra, and outline why it should control the derived deformation theory.

Malte Dehling University of Goettingen

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Pier Paolo La Pastina Sapienza Università di Roma

Deformations of VB-algebroids

A VB-algebroid is a vector bundle object in the category of Lie algebroids. I attach to every VB-algebroid a differential graded Lie algebra and I show that it controls deformations of the VB-algebroid structure. Several examples and applications are discussed. This is a joint work with Luca Vitagliano.

Takahiro Matsuyuki Tokyo Institute of Technology

Obstruction theoretical construction of characteristic classes of a fiber bundle

I shall introduce a construction of characteristic classes of a fiber bundle by simplicial method. We can get a certain obstruction class for a deformation of C_∞-algebra models of fibers and a characteristic map from the exterior algebra of a vector space of derivations. As applications, we can obtain known characteristic classes of surface bundles.
