Twisted Fock representation of Kähler manifolds and Hermitian-Einstein metrics from noncommutative U(1) instantons

Akifumi Sako

Department of Mathematics, Tokyo University of Science 1-3 Kagurazaka, Shinjuku-ku, Tokyo Japan

Joint work with: Kentaro Hara and Hyun Seok Yang

The Fock representation of noncommutative Kähler manifolds and its applications are discussed. Especially, Hermitian-Einstein metrics are constructed from noncommutative instantons on $\mathbb{C}^2$. Noncommutative Kähler manifolds studied here are constructed by deformation quantization with separation of variables. This deformation quantization was given by Karabegov. The algebra of the noncommutative Kähler manifolds contains the Heisenberg-like algebras. The algebras on noncommutative Kähler manifolds are represented as linear operators acting on the Fock space. Using the Fock representations, physical quantities in noncommutative Kähler manifolds are given by explicit functions. As an example of the application of the Fock representation, we make Hermitian-Einstein metrics. Hermitian-Einstein metrics are locally constructed by using self-dual two forms. As the self-dual two forms, $U(1)$ instantons on a noncommutative $\mathbb{C}^2$ are used here. To construct the noncommutative instantons on $\mathbb{C}^2$ we use the Fock representation. There is a dictionary between the basis of the Fock representations and ordinary functions on the Kähler manifolds. Using the dictionary concrete examples of Hermitian-Einstein metrics are obtained. This correspondence between the Hermitian-Einstein metrics and the noncommutative U(1) instantons is deeply related to the Seiberg-Witten map. Kähler conditions are concerned with the Bianchi identities for $U(1)$ gauge curvatures in commutative space.