Derived $\mathcal{D}$-Geometry

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Abstract

Derived $\mathcal{D}$-geometry is a combination of derived (or homotopical) algebraic geometry and algebraic $\mathcal{D}$-geometry. Homotopical algebraic geometry is algebraic geometry considered from the viewpoint of homotopical mathematics ('equalities replaced by homotopies'). Algebraic $\mathcal{D}$-geometry is the geometry of $\mathcal{D}_X$-schemes, i.e., of X-schemes equipped with an integrable connection along a smooth scheme X. More specifically, the opposite of the category of affine $\mathcal{D}_X$-schemes is the category of commutative unital algebras over the sheaf $\mathcal{D}_X$ of differential operators of X. One of the interesting concepts of derived $\mathcal{D}$-geometry is the notion of derived $\mathcal{D}_X$-stack, which we will explain using the functor of points approach. This type of stack appears in particular when dealing with bad intersections or quotients by badly behaved actions. Its most important aspect is the encoded action by ('total or horizontal') differential operators. The purpose of the talk is to provide evidence for derived $\mathcal{D}$-geometry being the convenient setting for a coordinate-free study of the moduli space of solutions of a system of PDEs modulo symmetries, in particular for the Batalin-Vilkovisky complex in gauge theories.