Julia sets with a wandering branching point

Jordi Canela
Université Paris-Est Marne-La-Vallée
Institut de Mathématiques de Toulouse

Joint Work with: **Xavier Buff** and **Pascale Roesch**

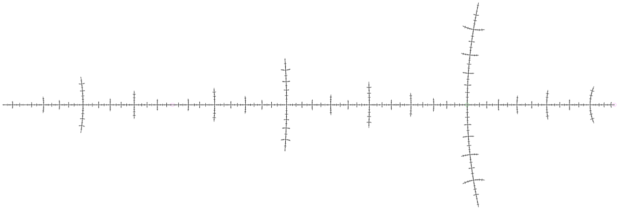
Będlewo, 23 March 2018

1. Branching points, wandering points, some results
2. A proof by perturbations
3. (k, ℓ)-configured polynomials
4. External rays of (k, ℓ)-configured polynomials
5. Wandering branching point: a sequence of perturbations

Branching point

Definition

Let $\mathcal{J}$ be a locally connected Julia set. Then, w is a *branching point* if $\mathcal{J} \setminus \{w\}$ has more than two components.



Wandering Point

The usual definition of wandering point is

Definition

A point is wandering if it admits a neighborhood U such that $f^n(U) \cap U = \emptyset$ for all $n \geqslant 0$.

With this definition no point in a Julia set $\mathcal{J}$ can be wandering, since for n large $f^n(U) \supset \mathcal{J}$.

Definition

We call a point in $\mathcal{J}$ *wandering* if it has an infinite orbit.

Wandering Point

The usual definition of wandering point is

Definition
A point is wandering if it admits a neighborhood U such that $f^n(U) \cap U = \emptyset$ for all $n \geqslant 0$.

With this definition no point in a Julia set $\mathcal{J}$ can be wandering, since for n large $f^n(U) \supset \mathcal{J}$.

Definition
We call a point in $\mathcal{J}$ *wandering* if it has an infinite orbit.

Wandering Point

The usual definition of wandering point is

Definition

A point is wandering if it admits a neighborhood U such that $f^n(U) \cap U = \emptyset$ for all $n \geqslant 0$.

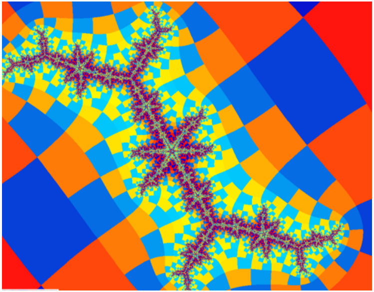
With this definition no point in a Julia set $\mathcal{J}$ can be wandering, since for n large $f^n(U) \supset \mathcal{J}$.

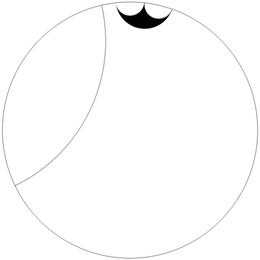
Definition

We call a point in $\mathcal{J}$ *wandering* if it has an infinite orbit.

No wandering triangle

Thurston (1985): A branch point of a locally connected Julia set of a quadratic polynomial P is either eventually periodic or eventually critical.





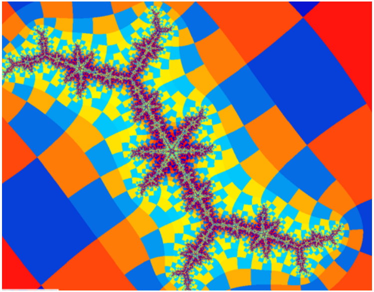
$\mathcal{J}(P)$ loc. con. $\Longrightarrow$ there is a lamination s.t. $\mathcal{J}(P) \simeq \overline{\mathbb{D}}/\sim$.

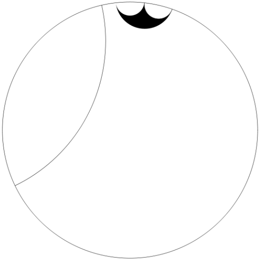
Thurston (1985): There is no wandering triangle in quadratic lamination.

Branching point $\leftrightarrow$ n-gon in the lamination with $n \geqslant 3$.

No wandering triangle

Thurston (1985): A branch point of a locally connected Julia set of a quadratic polynomial P is either eventually periodic or eventually critical.





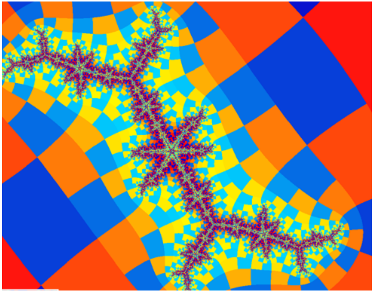
$\mathcal{J}(P)$ loc. con. $\Longrightarrow$ there is a lamination s.t. $\mathcal{J}(P) \simeq \overline{\mathbb{D}}/\sim$.

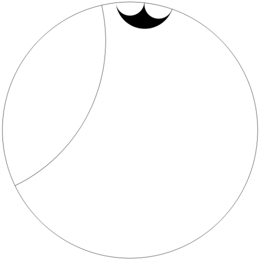
Thurston (1985): There is no wandering triangle in quadratic lamination.

Branching point $\leftrightarrow$ n-gon in the lamination with $n \geqslant 3$.

No wandering triangle

Thurston (1985): A branch point of a locally connected Julia set of a quadratic polynomial P is either eventually periodic or eventually critical.





$\mathcal{J}(P)$ loc. con. $\implies$ there is a lamination s.t. $\mathcal{J}(P) \simeq \overline{\mathbb{D}}/\sim$.
Thurston (1985): There is no wandering triangle in quadratic lamination.

Branching point $\leftrightarrow$ n-gon in the lamination with $n \geqslant 3$.

Wandering triangles: some results

- Thurston (1985):
  A branching point of a locally connected Julia set of a quadratic polynomial is either eventually periodic or eventually critical.
- Kiwi (2002):
  A wandering non pre-critical branching point of a degree d polynomial is the landing point of at most d external rays.
- Blokh (2005):
  If a cubic polynomial has wandering non-precritical branching points then the two critical points are recurrent one to each other.
- Blokh and Oversteegen (2008):
  There exist cubic polynomials with wandering non-precritical branching points.

Wandering triangles: some results

- Thurston (1985):
  A branching point of a locally connected Julia set of a quadratic polynomial is either eventually periodic or eventually critical.
- Kiwi (2002):
  A wandering non pre-critical branching point of a degree d polynomial is the landing point of at most d external rays.
- Blokh (2005):
  If a cubic polynomial has wandering non-precritical branching points then the two critical points are recurrent one to each other.
- Blokh and Oversteegen (2008):
  There exist cubic polynomials with wandering non-precritical branching points.

Wandering triangles: some results

- Thurston (1985):
  A branching point of a locally connected Julia set of a quadratic polynomial is either eventually periodic or eventually critical.
- Kiwi (2002):
  A wandering non pre-critical branching point of a degree d polynomial is the landing point of at most d external rays.
- Blokh (2005):
  If a cubic polynomial has wandering non-precritical branching points then the two critical points are recurrent one to each other.
- Blokh and Oversteegen (2008):
  There exist cubic polynomials with wandering non-precritical branching points.

Wandering triangles: some results

- Thurston (1985):
  A branching point of a locally connected Julia set of a quadratic polynomial is either eventually periodic or eventually critical.
- Kiwi (2002):
  A wandering non pre-critical branching point of a degree d polynomial is the landing point of at most d external rays.
- Blokh (2005):
  If a cubic polynomial has wandering non-precritical branching points then the two critical points are recurrent one to each other.
- Blokh and Oversteegen (2008):
  There exist cubic polynomials with wandering non-precritical branching points.

A concrete example

Buff-C.-Roesch: There exist a sequence of postcritically finite cubic polynomials $\{P_s\}$ converging to a cubic polynomial with wandering non-precritical branching points.

- We take a post-critical finite cubic polynomial such that:

  one critical point is iterated to the other and finally to a uniquely accessible fixed point.

- We construct a sequence of perturbations of the polynomial interchanging the critical dynamics.
- For each polynomial, some pre-critical point ξ_s separates 3 pre-periodic points.
- The sequence $\{\xi_s\}$ converges to a wandering non pre-critical branching point.

A concrete example

Buff-C.-Roesch: There exist a sequence of postcritically finite cubic polynomials $\{P_s\}$ converging to a cubic polynomial with wandering non-precritical branching points.

- We take a post-critical finite cubic polynomial such that:

  one critical point is iterated to the other and finally to a uniquely accessible fixed point.

- We construct a sequence of perturbations of the polynomial interchanging the critical dynamics.
- For each polynomial, some pre-critical point ξ_s separates 3 pre-periodic points.
- The sequence $\{\xi_s\}$ converges to a wandering non pre-critical branching point.

A concrete example

Buff-C.-Roesch: There exist a sequence of postcritically finite cubic polynomials $\{P_s\}$ converging to a cubic polynomial with wandering non-precritical branching points.

- We take a post-critical finite cubic polynomial such that:

  one critical point is iterated to the other and finally to a uniquely accessible fixed point.

- We construct a sequence of perturbations of the polynomial interchanging the critical dynamics.
- For each polynomial, some pre-critical point ξ_s separates 3 pre-periodic points.
- The sequence $\{\xi_s\}$ converges to a wandering non pre-critical branching point.

A concrete example

Buff-C.-Roesch: There exist a sequence of postcritically finite cubic polynomials $\{P_s\}$ converging to a cubic polynomial with wandering non-precritical branching points.

- We take a post-critical finite cubic polynomial such that:

  one critical point is iterated to the other and finally to a uniquely accessible fixed point.

- We construct a sequence of perturbations of the polynomial interchanging the critical dynamics.
- For each polynomial, some pre-critical point ξ_s separates 3 pre-periodic points.
- The sequence $\{\xi_s\}$ converges to a wandering non pre-critical branching point.

Set up

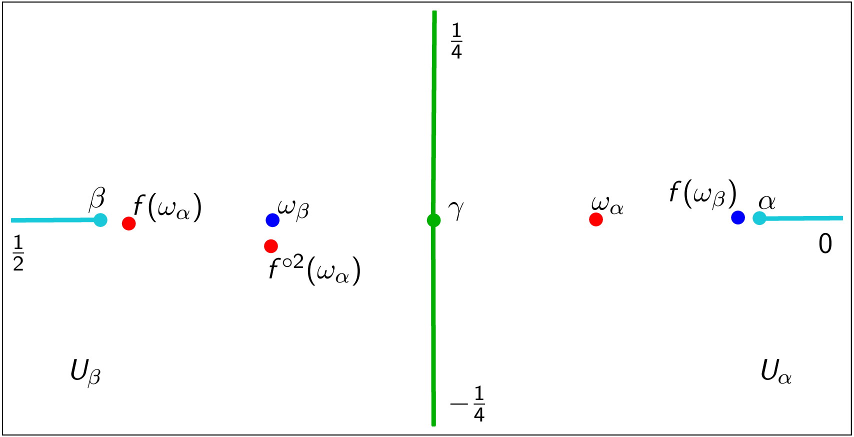
We consider the class $\mathfrak{V}$ of monic cubic polynomials P such that:

1. $P(0) = 0$;
2. P has three distinct repelling fixed point $\alpha = 0$, β and γ;
3. P has two distinct critical points ω_α, ω_β;
4. the ray of angle 0 lands at $\alpha = 0$;
5. the ray of angle $1/2$ lands at β;
6. the rays of angles $1/4$ and $-1/4$ land at γ, and separate the plane in two connected components
    - U_α containing α, ω_α and $f(\omega_\beta)$ and
    - U_β containing β, ω_β, $f(\omega_\alpha)$ and $f^{\circ 2}(\omega_\alpha)$.

Set up

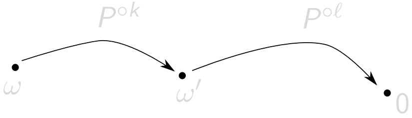
We consider the class $\mathfrak{V}$ of monic cubic polynomials P such that:

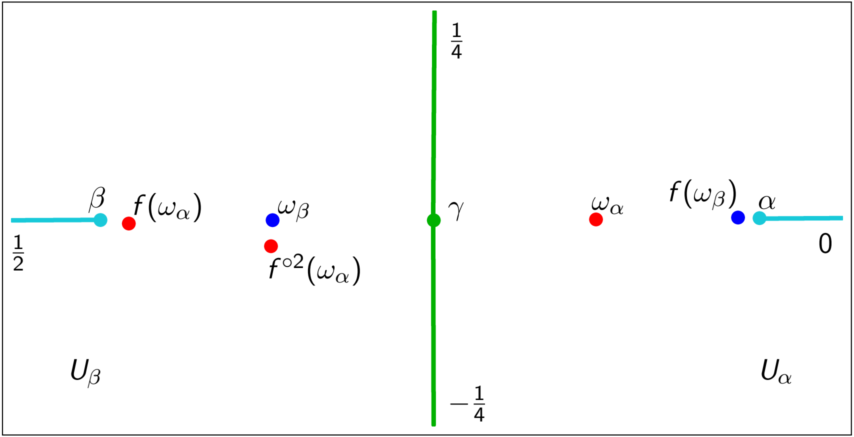
1. $P(0) = 0$;
2. P has three distinct repelling fixed point $\alpha = 0$, β and γ;
3. P has two distinct critical points ω_α, ω_β;
4. the ray of angle 0 lands at $\alpha = 0$;
5. the ray of angle $1/2$ lands at β;
6. the rays of angles $1/4$ and $-1/4$ land at γ, and separate the plane in two connected components
   - U_α containing α, ω_α and $f(\omega_\beta)$ and
   - U_β containing β, ω_β, $f(\omega_\alpha)$ and $f^{\circ 2}(\omega_\alpha)$.



We say that $P \in \mathfrak{V}$ has a (k, ℓ)-*configuration* if

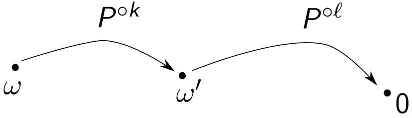
- there are $k, \ell > 0$ such that $P^{\circ k}(\omega) = \omega'$ and $P^{\circ \ell}(\omega') = 0$.





We say that $P \in \mathfrak{V}$ has a (k,ℓ)-*configuration* if

- there are $k, \ell > 0$ such that $P^{\circ k}(\omega) = \omega'$ and $P^{\circ \ell}(\omega') = 0$.



Perturbation of (k, ℓ)-configurations

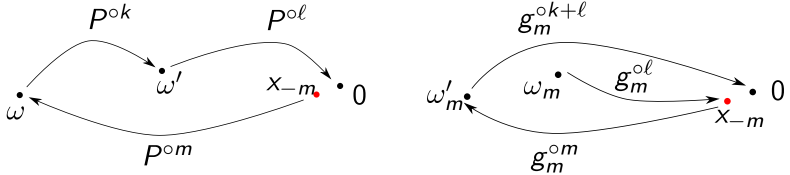
Lemma

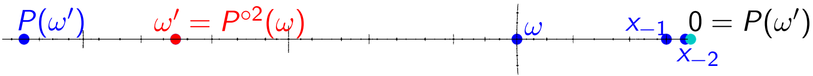
Assume $P \in \mathfrak{V}$ has a (k, ℓ)-configuration. Then, there are

- *a sequence $\{x_{-m}\}$ of preimages of ω,*
- *and sequence $\{g_m\}_{m \geqslant m_0}$ of polynomials in $\mathfrak{V}$ with critical points ω_m and ω'_m*

such that

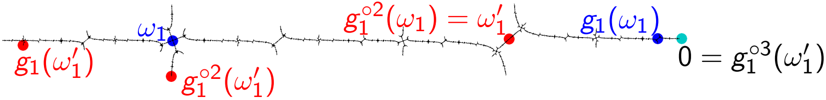
- $g_m \to P$;
- $\omega_m \to \omega', \quad \omega'_m \to \omega, \quad g_m^{\circ \ell}(\omega_m) = x_{-m}, \quad g_m^{\circ m}(x_{-m}) = \omega'_m$ *and* $g_m^{\circ k+\ell}(\omega_m) = \alpha$ *(g_m has an $(m+\ell, k+\ell)$-configuration).*





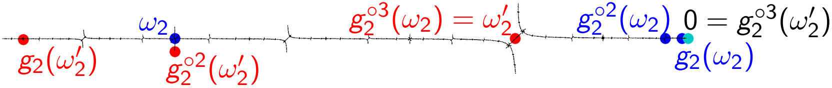
$P(\omega')$
$\omega' = P^{\circ 2}(\omega)$
ω
x_{-1}
x_{-2}
$0 = P(\omega')$

(c) P



ω_1
$g_1^{\circ 2}(\omega_1) = \omega_1'$
$g_1(\omega_1)$
$g_1(\omega_1')$
$g_1^{\circ 2}(\omega_1')$
$0 = g_1^{\circ 3}(\omega_1')$

(d) g_1



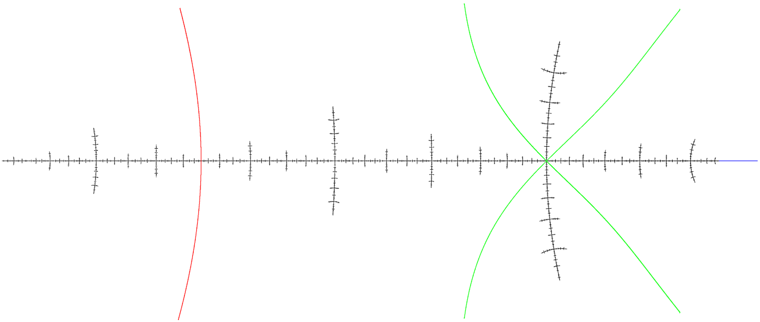
ω_2
$g_2^{\circ 3}(\omega_2) = \omega_2'$
$g_2^{\circ 2}(\omega_2)$
$0 = g_2^{\circ 3}(\omega_2')$
$g_2(\omega_2')$
$g_2^{\circ 2}(\omega_2')$
$g_2(\omega_2)$

(e) g_2

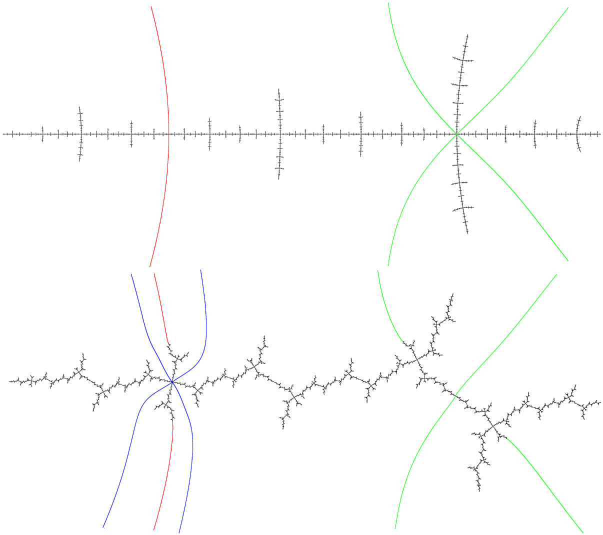
Pull back of the 0-ray at critical points

Assume that P has a (k, ℓ)-configuration.

- 0 is the landing point of a single external ray.
- $P^{\circ \ell}(\omega') = 0$. Hence, ω' is the landing point of 2 rays.
- $P^{\circ k}(\omega) = \omega'$. Hence, ω is the landing point of 4 rays.



After perturbation



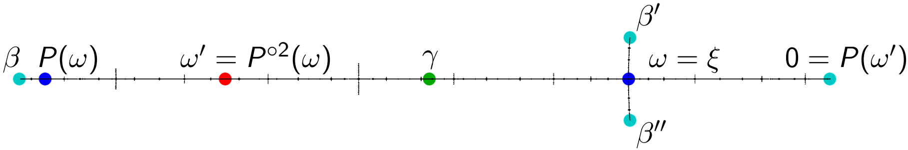
Admissible polynomials

Definition

A cubic polynomial $P \in \mathfrak{V}$ is *admissible* if it has critical points ω and ω' and a branching point ξ, such that:

1. ξ is precritical to ω, ω is precritical to ω' and ω' is prefixed to 0;
2. ξ separates β, β' and β'' in $\mathcal{J}(P)$, with $P^{-1}(\beta) = \{\beta, \beta', \beta''\}$.

We denote by j the integer such that $P^{\circ j}(\xi) = \omega$.



QUESTION: Is the perturbation of an admissible polynomial also admissible?

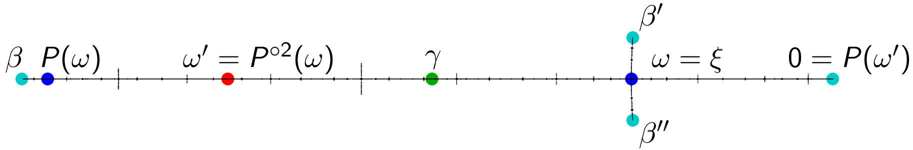
Admissible polynomials

Definition

A cubic polynomial $P \in \mathfrak{V}$ is *admissible* if it has critical points ω and ω' and a branching point ξ, such that:

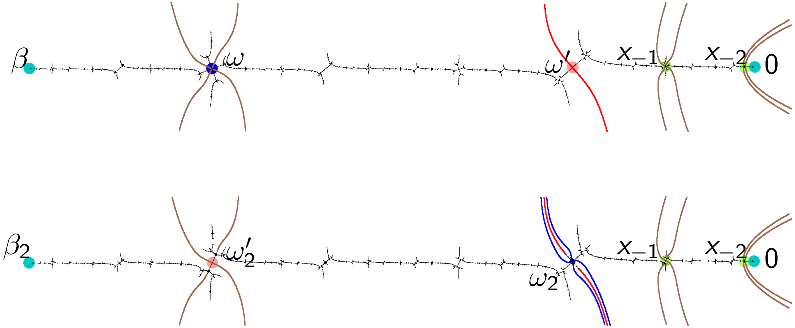
1. ξ is precritical to ω, ω is precritical to ω' and ω' is prefixed to 0;
2. ξ separates β, β' and β'' in $\mathcal{J}(P)$, with $P^{-1}(\beta) = \{\beta, \beta', \beta''\}$.

We denote by j the integer such that $P^{\circ j}(\xi) = \omega$.



QUESTION: Is the perturbation of an admissible polynomial also admissible?

If P is admissible, there is a sequence $\{x_h\}_{h\in\mathbb{N}}$ which accumulates at 0 'appropriately'.

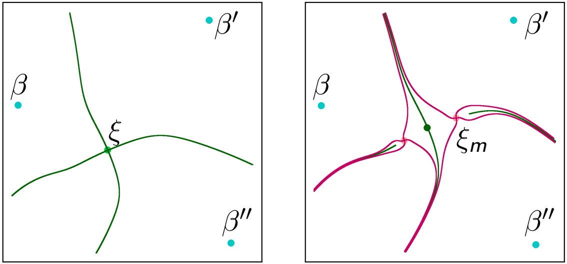


Separation of 3 points

Key lemma

There exists $m_0 > 0$ such that if $m > m_0$ then g_m is admissible:

- *there exists $\xi_m \to \xi$ separating β, β' and β'';*
- *ξ_m is mapped onto ω_m in $j + k$ iterates.*



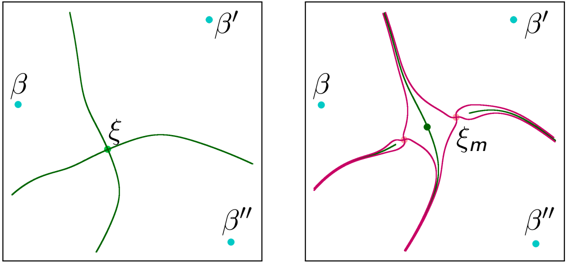
We use continuity of external rays after perturbation!

Separation of 3 points

Key lemma

There exists $m_0 > 0$ such that if $m > m_0$ then g_m is admissible:

- *there exists $\xi_m \to \xi$ separating β, β' and β'';*
- *ξ_m is mapped onto ω_m in $j + k$ iterates.*



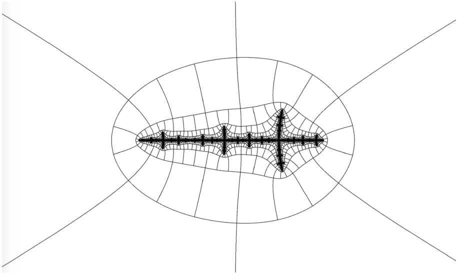
We use continuity of external rays after perturbation!

Convergence of Carathéodory loops

The continuous extension $\overline{\psi}$ of the inverse ϕ^{-1} of the Böttcher map ϕ restricts to a continuous map $\varphi : \mathbb{S}^1 \to \mathcal{J}(P)$ called the Carathéodory loop.

Proposition

Let $\{P_n\} \subset \mathfrak{V}$ be a sequence of postcritically finite polynomials which converge to an admissible cubic polynomial P. Then, the Carathéodory loops φ_n of P_n converge uniformly to the Carathéodory loop φ of P.

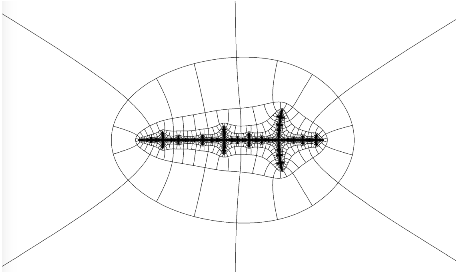


Convergence of Carathéodory loops

The continuous extension $\overline{\psi}$ of the inverse ϕ^{-1} of the Böttcher map ϕ restricts to a continuous map $\varphi : \mathbb{S}^1 \to \mathcal{J}(P)$ called the Carathéodory loop.

Proposition

Let $\{P_n\} \subset \mathfrak{V}$ be a sequence of postcritically finite polynomials which converge to an admissible cubic polynomial P. Then, the Carathéodory loops φ_n of P_n converge uniformly to the Carathéodory loop φ of P.



Iterative procedure

We begin with the following initial data:

- An admissible polynomial $P \in \mathfrak{V}$ with a (k, ℓ)-configuration.
- A branching point ξ, $P^{\circ j}(\xi) = \omega$, which separates β, β' and β''.
- A sequence $\{\epsilon_i\}$ of positive numbers, $\sum_{i=1}^{\infty} \epsilon_i = \epsilon$.

Set up

Set $P_0 = P$, $k_0 = k$, $\ell_0 = \ell$, $j_0 = j$.

- $P_0^{\circ k_0}(\omega_0) = \omega_0'$ and $P_0^{\ell_0}(\omega_0') = 0$.
- $P_0^{j_0}(\xi_0) = \omega_0$ and ξ_0 separates β, β' and β''.

We denote by φ_0 the Carathéodory loop of P_0.

Iterative procedure

We begin with the following initial data:

- An admissible polynomial $P \in \mathfrak{V}$ with a (k, ℓ)-configuration.
- A branching point ξ, $P^{\circ j}(\xi) = \omega$, which separates β, β' and β''.
- A sequence $\{\epsilon_i\}$ of positive numbers, $\sum_{i=1}^{\infty} \epsilon_i = \epsilon$.

Set up

Set $P_0 = P$, $k_0 = k$, $\ell_0 = \ell$, $j_0 = j$.

- $P_0^{\circ k_0}(\omega_0) = \omega_0'$ and $P_0^{\ell_0}(\omega_0') = 0$.
- $P_0^{j_0}(\xi_0) = \omega_0$ and ξ_0 separates β, β' and β''.

We denote by φ_0 the Carathéodory loop of P_0.

Step i

Let $\{g_{i,m}\}$ be a sequence of admissible perturbations of P_i with Carathéodory loops $\varphi(g_{i,m})$.
Fix m_i such that the following hold.

- $||P_i - g_{i,m_i}|| < \epsilon_i$.
- $||\varphi_i - \varphi(g_{i,m_i})|| < \epsilon_i$.

Let $P_{i+1} = g_{i,m_i}$ and let φ_{i+1} be its Carathéodory loop.

- P_{i+1} has critical points $\omega_{i+1} \approx \omega_i'$ and $\omega_{i+1}' \approx \omega_i$.
- $P_{i+1}^{\circ k_i + l_i}(\omega_{i+1}') = 0$.
- $P_{i+1}^{\circ l_i + m_i}(\omega_{i+1}) = \omega_{i+1}'$.
- There is a preimage ξ_{i+1} of ω_{i+1} which separates β, β' and β''. Moreover, $P_{i+1}^{\circ j_i + k_i}(\xi_{j+1}) = \omega_{i+1}$.

Set $j_{i+1} = j_i + k_i$, $k_{i+1} = \ell_i + m_i$ and $\ell_{i+1} = k_i + \ell_i$.

Step i

Let $\{g_{i,m}\}$ be a sequence of admissible perturbations of P_i with Carathéodory loops $\varphi(g_{i,m})$.
Fix m_i such that the following hold.

- $||P_i - g_{i,m_i}|| < \epsilon_i$.
- $||\varphi_i - \varphi(g_{i,m_i})|| < \epsilon_i$.

Let $P_{i+1} = g_{i,m_i}$ and let φ_{i+1} be its Carathéodory loop.

- P_{i+1} has critical points $\omega_{i+1} \approx \omega'_i$ and $\omega'_{i+1} \approx \omega_i$.
- $P_{i+1}^{\circ k_i + l_i}(\omega'_{i+1}) = 0$.
- $P_{i+1}^{\circ l_i + m_i}(\omega_{i+1}) = \omega'_{i+1}$.
- There is a preimage ξ_{i+1} of ω_{i+1} which separates β, β' and β''. Moreover, $P_{i+1}^{\circ j_i + k_i}(\xi_{j+1}) = \omega_{i+1}$.

Set $j_{i+1} = j_i + k_i$, $k_{i+1} = \ell_i + m_i$ and $\ell_{i+1} = k_i + \ell_i$.

Step i

Let $\{g_{i,m}\}$ be a sequence of admissible perturbations of P_i with Carathéodory loops $\varphi(g_{i,m})$.
Fix m_i such that the following hold.

- $||P_i - g_{i,m_i}|| < \epsilon_i$.
- $||\varphi_i - \varphi(g_{i,m_i})|| < \epsilon_i$.

Let $P_{i+1} = g_{i,m_i}$ and let φ_{i+1} be its Carathéodory loop.

- P_{i+1} has critical points $\omega_{i+1} \approx \omega'_i$ and $\omega'_{i+1} \approx \omega_i$.
- $P_{i+1}^{\circ k_i + l_i}(\omega'_{i+1}) = 0$.
- $P_{i+1}^{\circ l_i + m_i}(\omega_{i+1}) = \omega'_{i+1}$.
- There is a preimage ξ_{i+1} of ω_{i+1} which separates β, β' and β''. Moreover, $P_{i+1}^{\circ j_i + k_i}(\xi_{j+1}) = \omega_{i+1}$.

Set $j_{i+1} = j_i + k_i$, $k_{i+1} = \ell_i + m_i$ and $\ell_{i+1} = k_i + \ell_i$.

Step i

Let $\{g_{i,m}\}$ be a sequence of admissible perturbations of P_i with Carathéodory loops $\varphi(g_{i,m})$.
Fix m_i such that the following hold.

- $||P_i - g_{i,m_i}|| < \epsilon_i$.
- $||\varphi_i - \varphi(g_{i,m_i})|| < \epsilon_i$.

Let $P_{i+1} = g_{i,m_i}$ and let φ_{i+1} be its Carathéodory loop.

- P_{i+1} has critical points $\omega_{i+1} \approx \omega_i'$ and $\omega_{i+1}' \approx \omega_i$.
- $P_{i+1}^{\circ k_i + l_i}(\omega_{i+1}') = 0$.
- $P_{i+1}^{\circ l_i + m_i}(\omega_{i+1}) = \omega_{i+1}'$.
- There is a preimage ξ_{i+1} of ω_{i+1} which separates β, β' and β''. Moreover, $P_{i+1}^{\circ j_i + k_i}(\xi_{j+1}) = \omega_{i+1}$.

Set $j_{i+1} = j_i + k_i$, $k_{i+1} = \ell_i + m_i$ and $\ell_{i+1} = k_i + \ell_i$.

Step i

Let $\{g_{i,m}\}$ be a sequence of admissible perturbations of P_i with Carathéodory loops $\varphi(g_{i,m})$.
Fix m_i such that the following hold.

- $||P_i - g_{i,m_i}|| < \epsilon_i$.
- $||\varphi_i - \varphi(g_{i,m_i})|| < \epsilon_i$.

Let $P_{i+1} = g_{i,m_i}$ and let φ_{i+1} be its Carathéodory loop.

- P_{i+1} has critical points $\omega_{i+1} \approx \omega'_i$ and $\omega'_{i+1} \approx \omega_i$.
- $P_{i+1}^{\circ k_i + l_i}(\omega'_{i+1}) = 0$.
- $P_{i+1}^{\circ l_i + m_i}(\omega_{i+1}) = \omega'_{i+1}$.
- There is a preimage ξ_{i+1} of ω_{i+1} which separates β, β' and β''. Moreover, $P_{i+1}^{\circ j_i + k_i}(\xi_{j+1}) = \omega_{i+1}$.

Set $j_{i+1} = j_i + k_i$, $k_{i+1} = \ell_i + m_i$ and $\ell_{i+1} = k_i + \ell_i$.

Step i

Let $\{g_{i,m}\}$ be a sequence of admissible perturbations of P_i with Carathéodory loops $\varphi(g_{i,m})$.
Fix m_i such that the following hold.

- $||P_i - g_{i,m_i}|| < \epsilon_i$.
- $||\varphi_i - \varphi(g_{i,m_i})|| < \epsilon_i$.

Let $P_{i+1} = g_{i,m_i}$ and let φ_{i+1} be its Carathéodory loop.

- P_{i+1} has critical points $\omega_{i+1} \approx \omega'_i$ and $\omega'_{i+1} \approx \omega_i$.
- $P_{i+1}^{\circ k_i + l_i}(\omega'_{i+1}) = 0$.
- $P_{i+1}^{\circ l_i + m_i}(\omega_{i+1}) = \omega'_{i+1}$.
- There is a preimage ξ_{i+1} of ω_{i+1} which separates β, β' and β''. Moreover, $P_{i+1}^{\circ j_i + k_i}(\xi_{j+1}) = \omega_{i+1}$.

Set $j_{i+1} = j_i + k_i$, $k_{i+1} = \ell_i + m_i$ and $\ell_{i+1} = k_i + \ell_i$.

Limit map

- $P_i \to Q$, where Q is a monic cubic polynomial with $Q(0) = 0$.
- $\varphi_i \to \varphi$, where $\varphi : \mathbb{S}^1 \to \mathbb{C}$ continuous map.

Proposition

φ is the Carathéodory loop of Q ($\Rightarrow$ $\mathcal{J}(Q)$ is locally connected).

- Q has two critical points a and b which are wandering and recurrent one to the other.

Theorem

The branching points ξ_i converge to a point $\mathcal{Y}$ which is a wandering non-precritical branching point of Q.

Limit map

- $P_i \to Q$, where Q is a monic cubic polynomial with $Q(0) = 0$.
- $\varphi_i \to \varphi$, where $\varphi : \mathbb{S}^1 \to \mathbb{C}$ continuous map.

Proposition

φ is the Carathéodory loop of Q ($\Rightarrow$ $\mathcal{J}(Q)$ is locally connected).

- Q has two critical points a and b which are wandering and recurrent one to the other.

Theorem

The branching points ξ_i converge to a point $\mathcal{Y}$ which is a wandering non-precritical branching point of Q.

Limit map

- $P_i \to Q$, where Q is a monic cubic polynomial with $Q(0) = 0$.
- $\varphi_i \to \varphi$, where $\varphi : \mathbb{S}^1 \to \mathbb{C}$ continuous map.

Proposition

φ is the Carathéodory loop of Q ($\Rightarrow$ $\mathcal{J}(Q)$ is locally connected).

- Q has two critical points a and b which are wandering and recurrent one to the other.

Theorem

The branching points ξ_i converge to a point $\mathcal{Y}$ which is a wandering non-precritical branching point of Q.

Limit map

- $P_i \to Q$, where Q is a monic cubic polynomial with $Q(0) = 0$.
- $\varphi_i \to \varphi$, where $\varphi : \mathbb{S}^1 \to \mathbb{C}$ continuous map.

Proposition

φ is the Carathéodory loop of Q ($\Rightarrow$ $\mathcal{J}(Q)$ is locally connected).

- Q has two critical points a and b which are wandering and recurrent one to the other.

Theorem

The branching points ξ_i converge to a point $\mathcal{Y}$ which is a wandering non-precritical branching point of Q.

Limit map

- $P_i \to Q$, where Q is a monic cubic polynomial with $Q(0) = 0$.
- $\varphi_i \to \varphi$, where $\varphi : \mathbb{S}^1 \to \mathbb{C}$ continuous map.

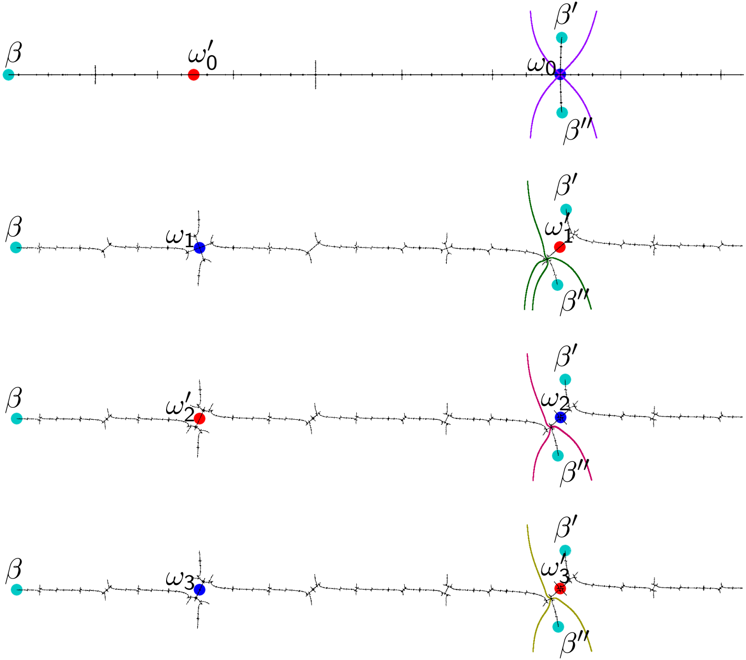
Proposition

φ is the Carathéodory loop of Q ($\Rightarrow \quad \mathcal{J}(Q)$ is locally connected).

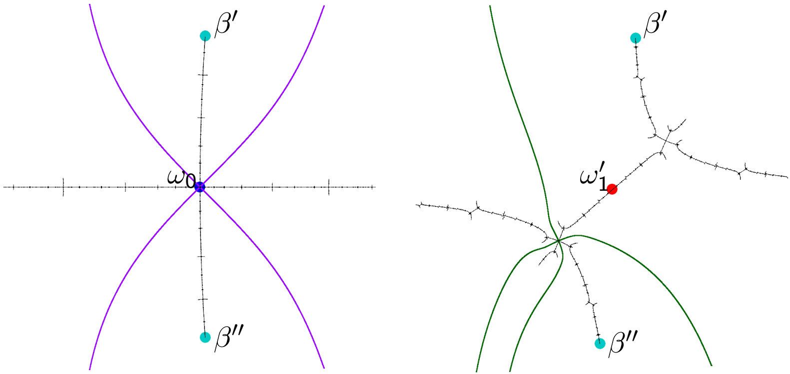
- Q has two critical points a and b which are wandering and recurrent one to the other.

Theorem

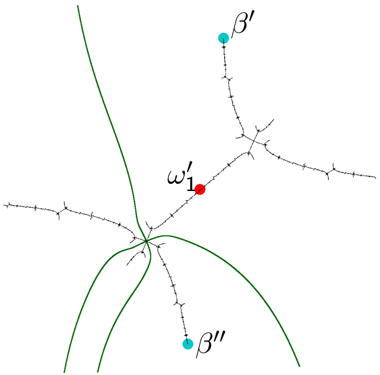
The branching points ξ_i converge to a point $\mathcal{Y}$ which is a wandering non-precritical branching point of Q.

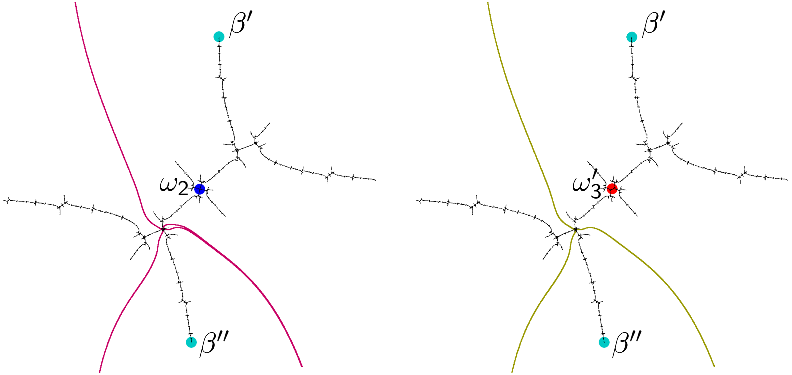


β
ω_0'
ω_0
β'
β''
β
ω_1
ω_1'
β'
β''
β
ω_2'
ω_2
β'
β''
β
ω_3
ω_3'
β'
β''



β'
ω_0
β''
β'
ω_1'
β''


β'
ω'_1
β''
β'
ω_2
β''



β′
ω₂
β″
β′
ω′₃
β″

Thank you very much for your attention!