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AFFINE SPACE FIBRATIONS ON ALGEBRAIC VARIETIES

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ABSTRACT. Let n be a positive integer and let X be an algebraic variety. An $\mathbb{A}^n$-fibration on X is a dominant morphism $f : X \longrightarrow Y$ such that f induces a regular extension $k(Y) \hookrightarrow k(X)$ of the function fields and general fibers of f are isomorphic to the affine n-space. It is the Dolgachev-Weisfeiler problem to ask if the generic fiber $X_\eta := X \times_Y \operatorname{Spec} k(Y)$ is isomorphic to $\mathbb{A}^n$ over the field $k(Y)$. If $n = 1, 2$ the answer is positive, but we do not know answers if $n \geq 3$. If $n = 1$ and $\dim X = 2$, the existence of $\mathbb{A}^1$-fibrations have played significant roles to clarify the structure of affine algebraic surfaces in affine algebraic geometry. Further, affine space fibrations are closely related to the quotient morphisms of algebraic unipotent group actions on X.

In the talk, we consider $\mathbb{A}^1$-fibrations ($\mathbb{A}^2$-fibrations as well) on affine threefolds with emphasis placed on the relations with $\mathbb{G}_a$-actions and singular fibers (characterizations of irreducible components, structure of two-dimensional components, locus of singular fibers on Y, etc.). Of particular interest is the existence of $\mathbb{A}^1$-fibrations in the complements of ample effective divisors in Fano threefolds.