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$\mathbb{A}^1$-CONTRACTIBILITY OF KORAS-RUSSELL LIKE VARIETIES

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Abstract. Koras-Russell threefolds first appeared in Koras and Russell's monumental work of showing that every $\mathbb{G}_m$-action on $\mathbb{C}^3$ is linearizable [KR97]. They are smooth, complex, topologically contractible, affine varieties of dimension three not isomorphic to $\mathbb{C}^3$ [ML96] which makes them potential counterexamples to the Zariski cancellation problem [Kra96]. In [HKØ16] it is shown that Koras-Russell threefolds of the first and second kind are stably $\mathbb{A}^1$-contractible and consequently $\mathbb{A}^1$-contractible after a finite suspension by the projective line $\mathbb{P}^1$- pointed at infinity. They are the first examples of smooth, complex, affine varieties not isomorphic to affine space which are stably $\mathbb{A}^1$-contractible. Koras-Russell threefolds of the first kind are in fact $\mathbb{A}^1$-contractible as shown in [DF15]. We extend the results in [HKØ16] and [DF15] to a bigger family of hypersurfaces in $\mathbb{C}^4$ including Koras-Russell threefolds of the first kind, threefolds with several degenerate fibers as well as hypersurfaces given by

$$x^n z = y^{\alpha_1} + t^{\alpha_2} + xq(x,t,y), \tag{1}$$

where $n, \alpha_1, \alpha_2 \geq 2$ and $\gcd(\alpha_1, \alpha_2) = 1$, and $q(x,y,t) \in \mathbb{C}[x,y,t]$ with $q(0,0,0) \neq 0$. The techniques in [HKØ16] and [DF15] can also be applied to hypersurfaces in $\mathbb{C}^4$ given by

$$x^n z = (x^m y + z^s)^{\alpha_1} + t^{\alpha_2} + x \tag{2}$$

with $n, m, s, \alpha_1, \alpha_2 \geq 2$ and $\gcd(s\alpha_1, \alpha_2) = 1$ obtained by an affine modification [KZ99] of a Koras-Russell threefold of the first kind. Additionally, we provide results on a four dimensional example, namely the hypersurface in $\mathbb{C}^5$ given by

$$x^n z = u^2 + t^3 + w^5 + x \tag{3}$$

with $n \geq 2$. The given examples (1), (2) and (3) are smooth, topologically contractible, complex, affine varieties not isomorphic to affine space which turn out to be stably $\mathbb{A}^1$-contractible, making them potential counterexamples to the Zariski cancellation problem.

References

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