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CLASSIFYING SMOOTH $\mathbb{Q}$-HOMOLOGY PLANES

TOMASZ PEŁKA

Polish Academy of Sciences

ABSTRACT. A $\mathbb{Q}$-homology plane ($\mathbb{Q}$HP) is a smooth affine surface S such that $H_i(S, \mathbb{Q}) = 0$ for $i > 0$. Such surfaces are well understood if $\kappa(S) < 2$, however, no structure theorems are known if $\kappa(S) = 2$. One of the main difficulties comes from the fact that in this case any smooth completion (X, D) of S is almost minimal, so the application of log MMP does not give much insight to its structure. To overcome this obstacle, K. Palka proposed to study the pair $(X, \frac{1}{2}D)$ instead. This approach leads to the Negativity Conjecture, which asserts that $\kappa(K_X + \frac{1}{2}D) = -\infty$, so a minimal model of $(X, \frac{1}{2}D)$ is a log Mori fiber space. This conjecture generalizes the Rigidity Conjecture of Flenner and Zaidenberg.

I will explain how to use log MMP for $(X, \frac{1}{2}D)$ to study of $\mathbb{Q}$HPs satisfying this conjecture. In particular, I will show that every such surface admits a $\mathbb{C}^{t*}$-fibration for some $t \leq 3$. I will also sketch the full classification in case when $S = \mathbb{P}^2 \setminus \bar{E}$ is a complement of a planar cuspidal curve. Eventually, I will indicate other directions where the log MMP with half-integral coefficients can be applied.

This is a joint work with Karol Palka.