Algebraic Geometry - Mariusz Koras in memoriam
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$\mathbb{C}^*$-ACTIONS ON $\mathbb{C}^3$ ARE LINEARIZABLE A SOMEWHAT HISTORICAL OVERVIEW OF THE PROOF AND ITS RAMIFICATIONS

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Abstract. I want to talk about the following, seemingly modest, result:

Let k be a field of characteristic zero. Then up to a choice of variables (x, y, z), a grading of the 3-dimensional polynomial algebra $k^{[3]}$ is defined by assigning integer weight (a, b, c) to the variables.

In more high-brow words, an algebraic action of the multiplicative group $G = \mathbb{G}_m$ of k on affine 3-space $X = \mathbb{A}^3$ is given by $t \cdot (x, y, z) = (t^a x, t^b y, t^c z)$ in suitable coordinates. This result has a surprising number of parents, Mariusz Koras prominent among them. It draws on a large number of results in algebraic geometry. For instance, already the fact that the fixed point set of the action is non-empty is non-trivial. Also, several new methods developed for the proof have inspired very nice new lines of investigation in related areas.

Let me add that the corresponding problem in dimensions 1 and 2 is much easier to resolve, but is open in dimensions 4 or higher, or in positive characteristic.

I will try to tell this story from a somewhat historical perspective, without becoming overly technical.