Canonical quantum gravity and the fate of singularities

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Motivation

- Singularity theorems in classical GR.
- What is the nature of the spacetime close to the high curvature regions?
- Can QG theories resolve all singularities?
- What are the observables we can measure?
- Can simple models provide predictions so that we can falsify them or even the full theory?

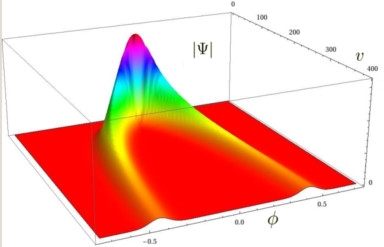
Motivation

- Symmetry reduced models allow us to realize concrete calculations (FRW, Bianchi, Schwarszchild, Kerr, ...).
- In quantum gravity, it is not obvious how to reduce the full quantum theory.
- The quantization of symmetry reduced models of GR can give us hints about the physics and mathematics of the full theory.
- For instance, we can study semiclassical sectors in agreement with GR and how quantum geometry can affect the predictions of the classical theory and its comparison with observations.

Homogeneous models in LQC

- Quantization of a FRW spacetime with a massless scalar field
  a) Quantum dynamics (improved scheme) ✓
  b) Singularity resolution (discrete quantum geometry) ✓
  c) Semiclassical sectors and effective dynamics ✓

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$



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- Extensions:

a) FRW with $k = 1$ ✓
b) FRW with $\Lambda = \pm 1$ ✓
c) Radiation dominated ✓
d) Bianchi I ✓
e) Bianchi II and IX
f) Kantowski-Sachs.

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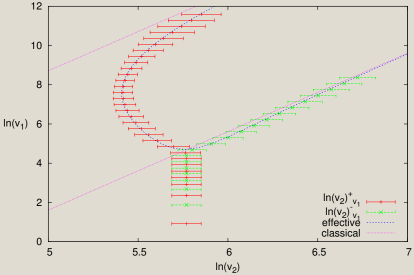

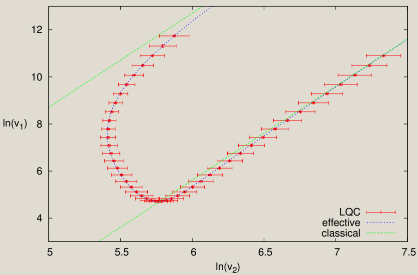

Martín-Benito, Mena Marugán, Pawlowski, Phys. Rev. D 80, 084038 (2009)

Inhomogeneous models

- Spherically symmetric vacuum spacetimes (Bojowald, Swiderski, 2004-2005) and polarized Gowdy models (Banerjee, Date, 2007).
  a) Symmetry reduction of the full theory in Ashtekar-Barbero variables.
  b) Robust kinematical quantum description.
  c) Well defined quantum Hamiltonian constraint (á la loop).
- Polarized Gowdy models – hybrid quantization (Garay, Martín-Benito, Mena-Marugán, 2008).
  a) Partial gauge fixing.
  b) Robust kinematical quantum description combining loop and Fock representation.
  c) Well defined quantum constraints (á la hybrid).
  d) Effective dynamics.

Inhomogeneous models

- Abelian constraint: Spherically symmetric spacetimes (Gambini, Pullin 2013; Gambini, O, Pullin 2013), (LRS) Gowdy cosmologies (Martín-de Blas, O, Pawlowski, 2015-2017) or 1+1 spacetimes like dilatonic black holes (Corichi, O, Rastgoo, 2016)

  a) classical abelianization of the Hamiltonian constraint,

$$H_T = \int dx(NH + H_r N^r),$$

$$H \to H_{\rm new} := \frac{(E^x)'}{E^\varphi} H - 2\frac{\sqrt{E^x}}{E^\varphi} K_\varphi H_r \Rightarrow \{H_{\rm new}(N), H_{\rm new}(\tilde{N})\} = 0.$$

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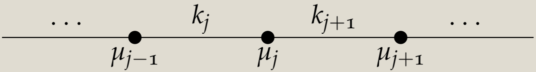
  a) classical abelianization of the Hamiltonian constraint,

  b) well-known kinematical Hilbert space,

$|g,\vec{k},\vec{\mu}\rangle =$



$$\Psi_{g,\vec{k},\vec{\mu}}(A_x, A_\varphi) = \prod_{e_j \in g} \exp\left(i k_j \int_{e_j} dx\, A_x(x) \right) \prod_{v_j \in g} \exp\left(i\mu_j A_\varphi(v_j) \right).$$

$$\langle g', \vec{k}', \vec{\mu}' | g, \vec{k}, \vec{\mu} \rangle = \delta_{g',g}\delta_{\vec{k}',\vec{k}}\delta_{\vec{\mu}',\vec{\mu}}.$$

Inhomogeneous models

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  a) classical abelianization of the hamiltonian constraint
  b) well-known kinematical Hilbert space
  c) Physical Hilbert space

  $$\|\Psi_{phys}\|^2 = \left(\langle\Psi_{kin}|\eta_{diff(H_r)}\eta_{diff(H)}\right)|\Psi_{kin}\rangle$$

  and Dirac observables

  $$\hat{O}(z) = \ell_{\rm Pl}^2\hat{k}_{{\rm Int}(nz)},$$

  and one physical global degree of freedom (either the mass $\hat{M}$ or a densitized shear scalar $\hat{h}$), are known.

Black hole spacetimes

- The effective spacetime metric can be computed by means of parametrized (relational) observables. For instance, in the spatially flat gauge $ds^2 = d\tilde{s}^2 + \langle \hat{E}^x(x) \rangle d^2\Omega$, where

$$d\tilde{s}^2 = -\langle \left(1 - \frac{\hat{r}_S}{\sqrt{\hat{E}^x(x)}} \right) \rangle dt^2 - \langle \eta \sqrt{\frac{\hat{r}_S}{[\hat{E}^x(x)]^{3/2}}} [\hat{E}^x(x)]' \rangle dt dx + \langle \left(\frac{[\hat{E}^x(x)]'}{4\hat{E}^x(x)} \right)^2 \rangle dx^2$$

- These geometries are discrete (piecewise constant x-functions).
- At high curvature regions the effective geometries are regular (singularity free).
- Interplay between the (fluctuating) discrete geometry and the (fluctuating) horizon (?).

Gowdy cosmologies with local rotational symmetry

- These classical geometries are diffeomorphic to vacuum Bianchi I spacetimes with local rotational (axial) symmetry (Dey, Kovetz, Paban - 2012, 2014).
- The phase space variables are $(\mathcal{A}, \mathcal{E})$ and (A_x, E^x). The components of the classical spatial metric are $g_{\theta\theta} = (E^x)^2\mathcal{E}^{-1}$ and $g_{xx} = \mathcal{E} = g_{yy}$.
- It is possible to follow the quantization strategy of spherically symmetric spacetimes – discreteness, singularity resolution, etc. (Martín-de Blas, O, Pawlowski - 2015).

Gowdy cosmologies with local rotational symmetry

- We also showed that it is possible to implement an improved dynamics scheme with $E^x \to V = \sqrt{\mathcal{E}} E^x$.
- On the physical Hilbert space, the quantum Hamiltonian evolution is defined as follows
  - Choice of phase space variable as time function T_j (on v_j).
  - A family of unitary (norm preserving) transformations $\hat{P}_{T_j}$ between $\mathcal{H}_{\rm phy}$ and $\mathcal{H}_T$.
  - Relevant operators (observables) $\{\hat{O}(T)\}$ with suitable domains in $\mathcal{H}_T$.
- Finally, the evolution is defined via a family of operators $|\psi_{T'}\rangle := \hat{U}_{T',T}|\psi_T\rangle = \hat{P}_{T'}(\hat{P}_T)^{-1}|\psi_T\rangle$.
- As an example, we choose $V(\theta)$ as time function (non monotonic). Evolution split on several charts (on each vertex) (Martín-de Blas, O, Pawlowski - 2017).

Gowdy cosmologies with local rotational symmetry

- The solutions to the constraint and inner physical product can be written as

$$\tilde{\Psi}(\vec{k},h,\vec{v}) = \tilde{\Psi}(\vec{k},h)e_{\vec{k},h}(\vec{v}), \quad \langle\Phi|\Psi\rangle = \sum_{\vec{k}\in(\mathbb{Z}^\star)^n} \int_0^{h_\star(\vec{k})} dh \tilde{\Phi}^\star(\vec{k},h)\tilde{\Psi}(\vec{k},h).$$

- We split the solutions as via $\Psi^\pm(v) = \mathcal{F}^{-1}\theta(\pm b)[\mathcal{F}\Psi](b)$. This defines the unitary maps $\hat{U}^\pm_{\vec{v}_2,\vec{v}_1} = \hat{P}^\pm_{\vec{v}_2}(\hat{P}^\pm_{\vec{v}_2})^{-1}$ with

$$\tilde{\Psi}^\pm_{\vec{v}}(\vec{k},h) = \hat{P}^\pm_{\vec{v}}\tilde{\Psi}(\vec{k},h) = \frac{e^\pm_{\vec{\omega}(\vec{k},h)}(\vec{v})}{|e^\pm_{\vec{\omega}(\vec{k},h)}(\vec{v})|}\tilde{\Psi}(\vec{k},h).$$

- The evolution on each chart, namely volume expanding and contracting, (in the Schrödinger picture)

$$|\Psi^\pm_{\vec{v}}\rangle = \sum_{\vec{k}\in(\mathbb{Z}^\star)^n} \int_0^{h_\star(\vec{k})} dh \tilde{\Psi}(\vec{k},h)\frac{e^\pm_{\vec{\omega}(\vec{k},h)}(\vec{v})}{|e^\pm_{\vec{\omega}(\vec{k},h)}(\vec{v})|}|\vec{k},h\rangle, \quad \langle\Psi^\pm_{\vec{v}}|\Psi^\pm_{\vec{v}}\rangle = 1, \quad \forall\vec{v}.$$

- Dirac observables $\quad \hat{O}_{\vec{v}'} = \hat{U}^\pm_{\vec{v}',\vec{v}}\hat{O}_{\vec{v}}(\hat{U}^\pm_{\vec{v}',\vec{v}})^{-1}$

Gowdy cosmologies with local rotational symmetry

- Merits:
  - Rigorous quantum Hamiltonian evolution (asymptotically).
  - It is not necessary to rely on the classical theory.
- Limitations:
  - The time-dependent states are not solutions to the constraint (but are related with them via a bijection).
  - The observables have ambiguous physical meaning close to the turning points (there it is more convenient to switch the time function).
  - Not obvious application to phase space variables with multiple turning points.
- Prospects: Extension of this Hamiltonian evolution in the context of black hole spacetimes.

Quantum field theories on quantum geometries

- Test fields (no backreaction) on these quantum geometries (perturbations) experience a dressed effective geometry.

$$\int d^4x \mathcal{L}_\phi^{\text{class}} = \frac{1}{2}\int dt d^3x \sqrt{-g}\left[-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m_\phi^2\phi^2\right] \rightarrow$$
$$\int d^4x \mathcal{L}_\phi^{\text{dress}} = \frac{1}{2}\int dt d^3x \left[-\langle\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\rangle\partial_\mu\phi\partial_\nu\phi - \langle\sqrt{-\hat{g}}\rangle m_\phi^2\phi^2\right]$$

- The propagation of fields on these (inhomogeneous) quantum geometries is not well understood.

- Extraction of predictions: role of discrete geometry and quantum anisotropies on the CMB; Hawking radiation, spectrum of quasi-normal modes (O - 2018), ...

QFT on quantum BHs

- For instance, the effective Hamiltonian of a test field (with parametrized observables in spatially flat gauge coordinates)

$$\hat{\mathcal{H}}_\psi^{\rm sph} = \langle\psi|\hat{\mathcal{H}}^{\rm sph}|\psi\rangle = \int dx \sum_{\ell m} \frac{1}{2}\Big(\mathcal{P}_1 \hat{\pi}^*_{\ell m}\hat{\pi}_{\ell m} + \mathcal{P}_2 \partial_x\hat{\phi}^*_{\ell m}\partial_x\hat{\phi}_{\ell m} + \Big(\mathcal{P}_3 m_\phi^2 + \mathcal{P}_4 \ell(\ell+1)\Big)\hat{\phi}^*_{\ell m}\hat{\phi}_{\ell m}\Big) + \hat{\pi}^*_{\ell m}\mathcal{P}_5\partial_x\hat{\phi}_{\ell m},$$

$$\mathcal{P}_1(x) = \langle\psi|\left[\widehat{\frac{2}{(E^x)'\sqrt{E^x}}}\right]|\psi\rangle, \quad \mathcal{P}_2(x) = \langle\psi|\left[\frac{\widehat{2\sqrt{E^x}E^x}}{(E^x)'}\right]|\psi\rangle, \quad \mathcal{P}_3(x) = \langle\psi|\left[\frac{\widehat{(E^x)'\sqrt{E^x}}}{2}\right]|\psi\rangle$$

$$\mathcal{P}_4(x) = \langle\psi|\left[\widehat{\frac{(E^x)'}{2\sqrt{E^x}}}\right]|\psi\rangle, \quad \mathcal{P}_5(x) = \langle\psi| - 2\eta\sqrt{2\hat{M}}\left[\frac{\widehat{(E^x)^{1/4}}}{(E^x)'}\right]|\psi\rangle.$$

$$\tilde{N}^2 = \mathcal{P}_4\sqrt{\mathcal{P}_1\mathcal{P}_2}, \quad \tilde{N}^x = \mathcal{P}_5, \quad \tilde{q}_{xx} = \frac{\mathcal{P}_4}{\sqrt{\mathcal{P}_1\mathcal{P}_2}}, \quad \tilde{q}_{\theta\theta} = \sqrt{\frac{\mathcal{P}_2}{\mathcal{P}_1}} = \frac{\tilde{q}_{\varphi\varphi}}{\sin^2\theta}, \quad \tilde{m}_\phi^2 = m_\phi^2\sqrt{\frac{\mathcal{P}_1}{\mathcal{P}_2}}\frac{\mathcal{P}_3}{\mathcal{P}_4}.$$

Summary

- Loop quantum gravity techniques applied to symmetry reduced models successfully deals with several fundamental questions: singularity resolution, semiclassical geometries, effective description, etc.
- Full quantization of inhomogeneous cosmological and black hole models is available.
- Their quantum dynamics, semiclassical sectors and effective geometries need to be further explored.
- Test quantum fields on these quantum geometries deserve additional attention (dressed effective metric): the fundamental discretization and fluctuations of these quantum geometries can modify our understanding of several well-known phenomena of QFTs on classical curved spacetimes. Further research in this direction seems very promising.