BLACK HOLES INFORMATION FLUX

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OUTLINE

- On burning a lump of coal: standard entropy budget in thermodynamic
- Coarse-graining the entropy
- Extension to black holes: Entropy/Information flux in Hawking radiation
  - Thermodynamic entropy
  - Entanglement entropy
    - Bipartite system
    - Tripartite system
    - Multipartite system
- Quantum corrections to the Hawking flux
- Discussion

ON BURNING A LUMP OF COAL

- Standard statistical mechanics applied to a furnace with a small hole ⟶ blackbody radiation (Planck spectrum implies some coarse graining)
  - Transfer of thermodynamic entropy to the radiation field: $S = \frac{E}{T} = \frac{\hbar\omega}{T}$
- Consider the effect of coarse graining the (von Neumann) entropy

$$S_{coarse\ grained} = S_{before\ coarse\ graining} + S_{correlations} \longrightarrow I_{correlations}$$

- The average energy per photon: $\langle E \rangle = \hbar \langle \omega \rangle = \hbar \frac{\int \omega f(\omega) d\omega}{\int f(\omega) d\omega} = \frac{\pi^4}{30\zeta(3)} k_B T$
- Consequently, we can calculate the average entropy per photon (and the standard deviation) [A. A-S, Matt Visser, PLB 757 (2016) 383)]

$$\boxed{\hat{S}_2 = 3.9 \pm 2.5} \text{ bits/photon}$$

- Since we know the underlying physics is unitary ⟶ this entropy is compensated by an equal "hidden information"

COARSE GRAINING THE ENTROPY

- Explicit and calculable processes that takes an arbitrary system and monotonically and controllably drives the entropy to the maximum value
- It is relevant when we deal with physical issues of information
  - Price of coarse graining: some properties are hidden in the system
  - Measure of uncertainty → entropy as a lack of information

<u>Coarse graining the classical Shannon entropy</u>

- Continuum → continuum: Diffusion process
- Continuum → discretium: Box averaging
- Discretium → discretium: Aggregation/averaging

<u>Coarse graining the quantum Von Neumann entropy</u>

- Maximal mixing
- Tunable Partial traces
- Full/tunable partial decoherence

EXTENSION TO BLACK HOLES

★ unitarity ★ apparent/trapping horizons ★ complete evaporation

THERMODYNAMIC ENTROPY IN THE HAWKING FLUX

- Loss of Bekenstein entropy of a Schwarzschild black hole

$$\frac{dS}{dN}=\frac{dS/dt}{dN/dt}=\frac{d\left(4\pi k_B GM^2/\hbar c\right)/dt}{dN/dt}=\frac{k_B\pi^4}{30\zeta(3)}$$

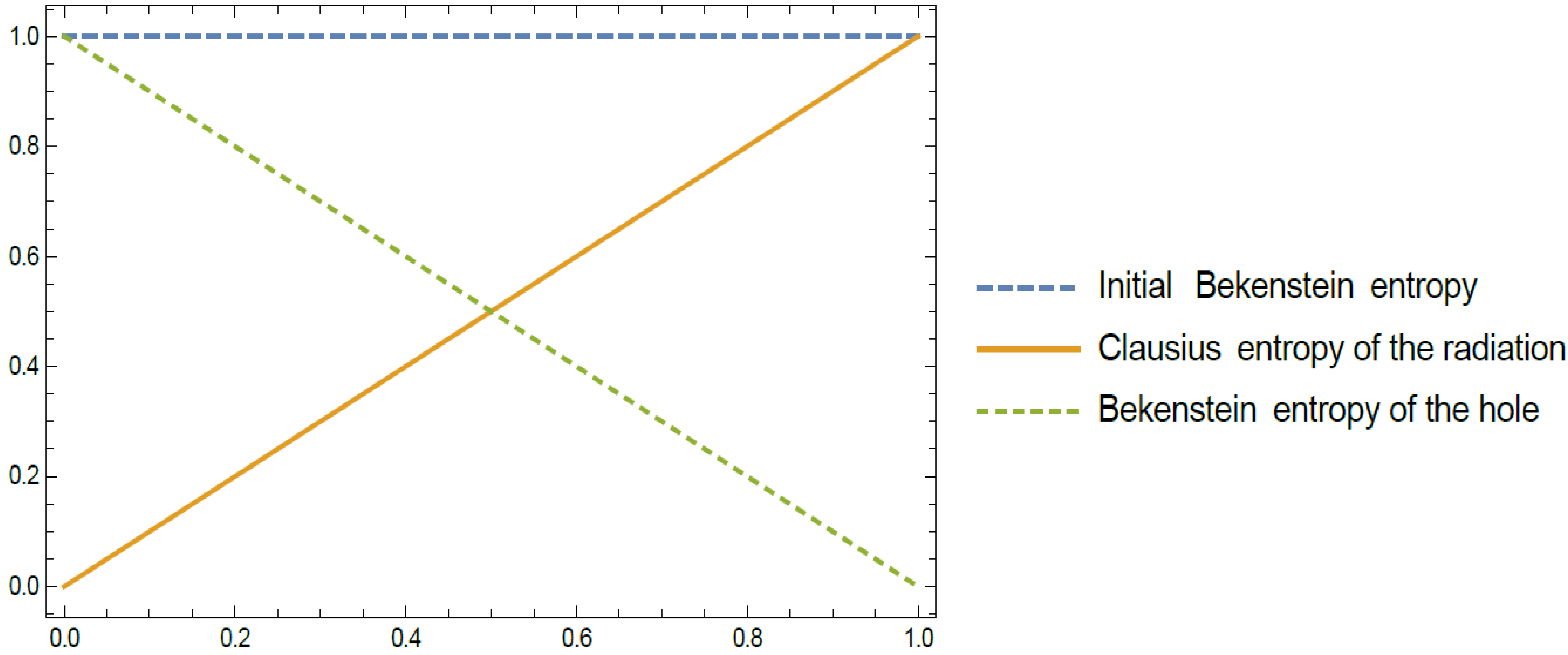
- If we measure the entropy in bits: $\frac{d\hat{S}_2}{dN}\approx 3.896976$ bits/quanta

- The Hawking radiation is essentially (adiabatically) transferring Bekenstein entropy from the hole into Clausius entropy of the radiation field
  [A. A-S, M. Visser, PLB 776 (2018) 10-16]

- Estimation of the total number of emitted massless quanta

$$\frac{dN}{dM}=\frac{30\zeta(3)}{\pi^4}\frac{8\pi GM}{\hbar c}$$

- Throughout the evaporation process we have $S_{Bekenstein}(t)+S_{Clausius}(t)=S_{Bekenstein,0}$



- So, semiclassically everything holds together very well

- Sparsity of the Hawking flux: Average time between emission of successive Hawking quanta is many times larger than the natural timescale set by the energies of the emitted quanta [F. Gray et al., CQG 33 (2016) 115003]

ENTANGLEMENT ENTROPY IN THE HAWKING FLUX

- Average subsystem entropies [D. N. Page, PRL 71(1993) 1291]

- Consider a Hilbert space that factorizes $H_{AB} = H_A \otimes H_B$

  Pure state $\rho_{AB} = |\psi\rangle\langle\psi|$ ⟶ Subsystem density matrices $\rho_A = tr_B(|\psi\rangle\langle\psi|)$

  Subsystem von Neumann entanglement entropy $\hat{S}_A = -tr(\rho_A \ln \rho_A)$

- Uniform average over all pure states, taking:

  $n_1 = dim(H_A)$, $n_2 = dim(H_B)$ and $m = min[n_{1,} n_2]$

  The central result $\hat{S}_{n_{1,} n_2} = \langle \hat{S}_A \rangle = \langle \hat{S}_B \rangle \leqslant \ln m$

- Average subsystem entropy is very close to its maximum possible value

  Strict bound (combined with our results): $\hat{S}_{n_{1,} n_2} = \langle \hat{S}_A \rangle = \langle \hat{S}_B \rangle \in \left(\ln m - \frac{1}{2}, \ln m \right)$

Bipartite entanglement: black hole + Hawking radiation

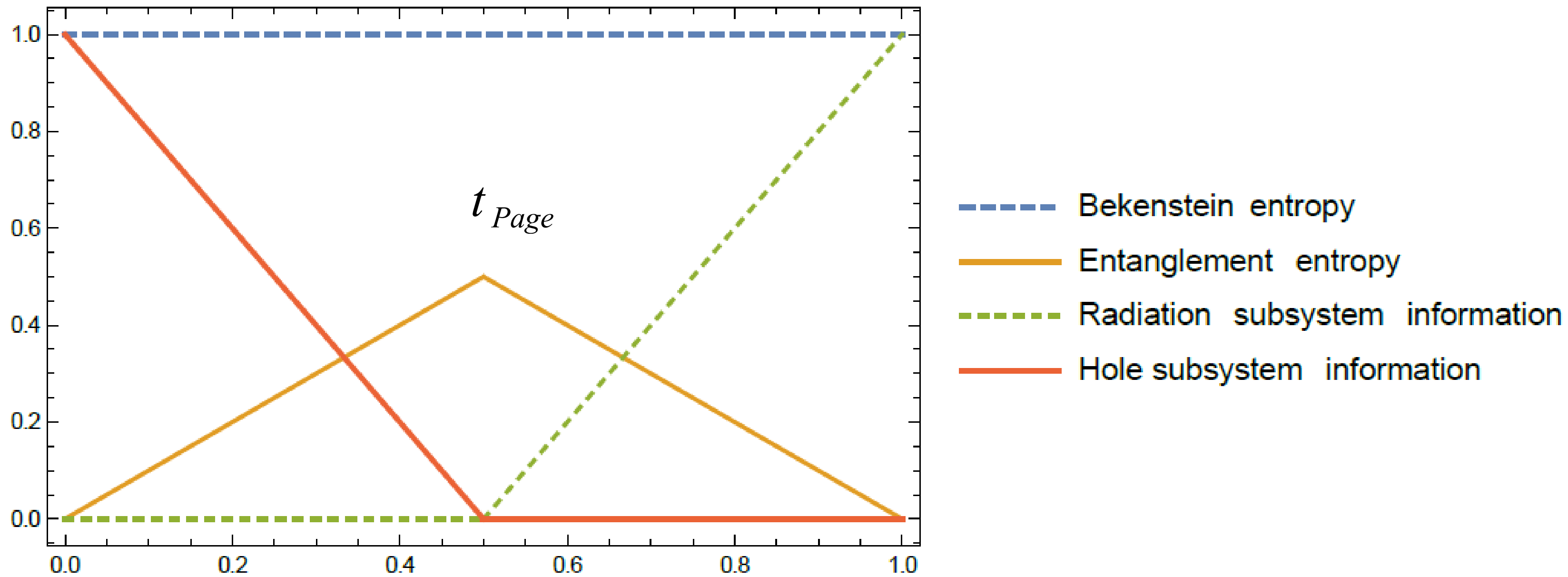
[D. N. Page, PRL 71(1993) 3743]

- "Closed box" argument
- Initially there is no yet any Hawking radiation $\left\{ \begin{array}{ll} H_R & \text{trivial} \\ H_H & \text{enormous} \end{array} \right. \longrightarrow (\hat{S}_{n_H, n_R})_0 = 0$
- After the black hole has completely evaporated: H_H is trivial $\longrightarrow (\hat{S}_{n_H, n_R})_\infty = 0$
- At intermediate times both dimensionalities are nontrivial $\longrightarrow (\hat{S}_{n_H, n_R})_t \neq 0$
  - Since the evolution is assumed unitary: total Hilbert space is constant

$$n_H(t) n_R(t) = n_{H_0} = n_{R_\infty} \longrightarrow (\hat{S}_{n_H, n_R})_t = \ln \left\{ n_H(t), \frac{n_{H_0}}{n_H(t)} \right\}$$

Maximized when $n_H(t) \approx \sqrt{n_{H_0}} \longrightarrow \hat{S}_{n_H, n_R}(t = t_{Page}) \approx \frac{1}{2} \ln n_{H_0}$

➢ Page curve:



- Pages defines a novel asymmetric version of the subsystem information (no direct physical interpretation) → Mutual information and other measures of entanglement (such as negativity, tangle or concurrence)

- It is the shape of this curve that underlies much of the modern discussion surrounding the "information puzzle"

- Subsystem entropy is initially zero → tension with Bekenstein entropy

- If we entangle the black hole with the environment, then the total state is not pure

- What happens with these results?

  - The late radiation is maximally entangled with the early radiation and the hole subsystems

⬇

Monogamy of entanglement

⬇

Firewalls

- We though that the point is the considered “closed box” system

  - May be it is more appropriate to consider a tripartite system, including the interaction with the environment

Tripartite entanglement: bh + Hawking radiation + rest of the universe

[A. A-S, Matt Visser, PLB 757 (2016) 383)]

- The Hilbert space is now $H_{HRE} = H_H \otimes H_R \otimes H_E$
- Take the entire universe be in a pure state $S_{HRE}(t) = 0$

  And now the subsystem entropies $S_H(t) = S_{ER}(t)$, $S_R(t) = S_{HE}(t)$, $S_E(t) = S_{HR}(t)$
- Initially $S_{H_0} = S_{E_0}$, $S_{R_0} = 0 = S_{HE_0}$
- Once the black hole has completely evaporated $S_{H_\infty} = 0 = S_{ER_\infty}$, $S_{R_\infty} = S_{E_\infty}$
- The evolution is assumed unitary, with the unitary time evolution operator factorized as

$$U_{HRE} = U_{HR}(t) \otimes U_E(t)$$

  Therefore $\begin{cases} n_{E_0} = n_{E_\infty} \equiv n_E \\ n_H(t)\, n_R(t) = n_{H_0} = n_{R_\infty} \end{cases}$

- We make an additional assumption: That the Bekenstein entropy can be identified with the average entanglement entropy

➢ Then, the entropies:

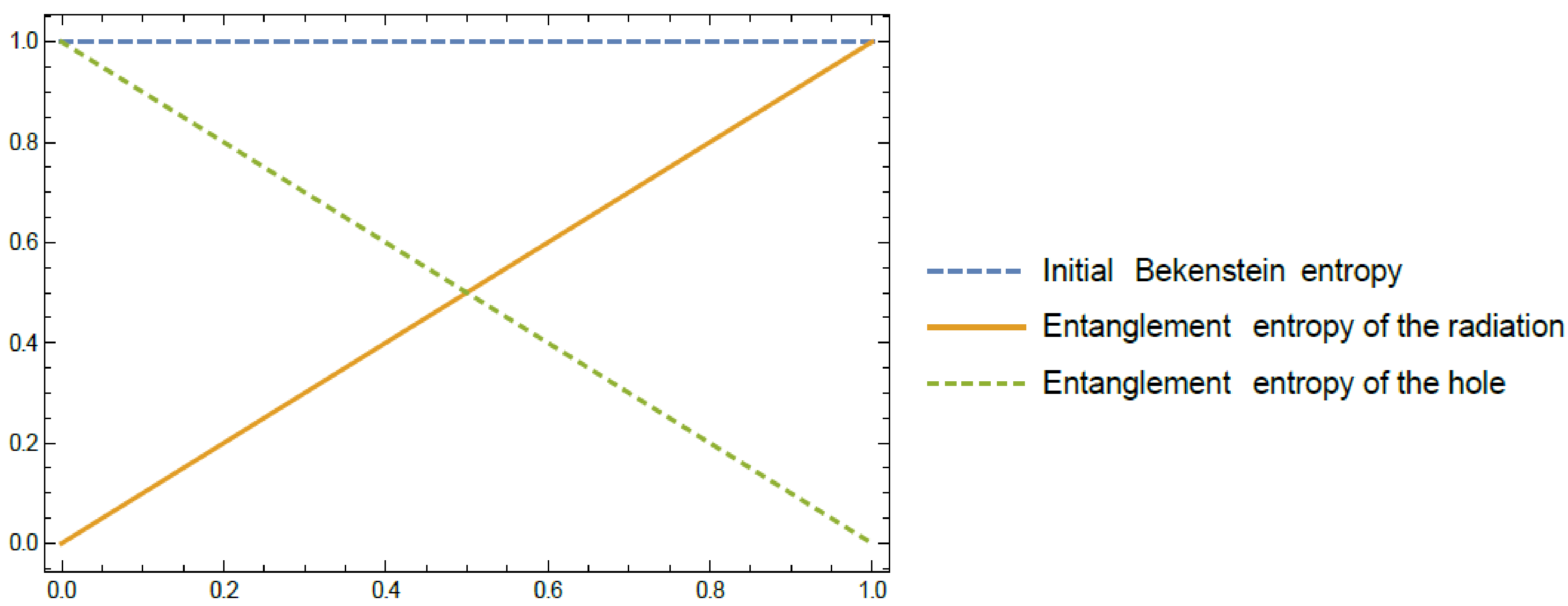
- $\left\langle \hat{S}_H(t) \right\rangle \approx \ln min[n_H(t), n_R(t) n_E] \approx \ln n_H(t)$
- $\left\langle \hat{S}_R(t) \right\rangle \approx \ln min[n_R(t), n_H(t) n_E] \approx \ln n_R(t)$

$$\left\langle \hat{S}_H(t) \right\rangle + \left\langle \hat{S}_R(t) \right\rangle \approx \ln[n_H(t) n_R(t)] = \ln n_{H_0}$$

$$\hat{S}_{Bekenstein}(t) + \left\langle \hat{S}_{Hawking\ radiation}(t) \right\rangle \approx \hat{S}_{Bekenstein,0}$$

➢ The "rest of the universe environment": the extent to which the subsystem is entangled

$$\left\langle \hat{S}_E(t) \right\rangle \approx \ln min[n_E(t), n_H(t) n_R] \approx \ln n_{H_0} \approx \hat{S}_{Bekenstein,0}$$



Mutual information

- For the tripartite system: $I_{H:R} = S_H + S_R - S_{HR}$ → S_E

- Averaging over the pure states in the total system we obtain

$$\left\langle \hat{I}_{H:R} \right\rangle \leqslant \frac{n_{H_0}}{2\mathrm{n}_E} \leqslant \frac{1}{2}$$

➡ So the average mutual information never exceeds ½ nat throughout the entire evaporation process

Multi-partite entanglement

- The Hilbert space is now $H = \otimes_{i=1}^{N} H_i$

- Then, the partial traces are defined as

$$\rho_{ij} = tr_{H/H_i \otimes H_j} \rho$$

- The mutual information is given by

$$\left\langle \hat{I}_{(ijk...):(pqr...)} \right\rangle \leqslant \frac{n_{ijk...}^2 \, n_{pqr...}^2}{2\text{n}} \leqslant \frac{1}{2}$$

➡ So the average mutual information between any two "small" collections of subsystems in the multi-partite pure-state system never exceeds ½ nat as long as the rest of subsystem collection is dominant
[A. A-S, Matt Visser, PRA 96 (2017) 052302]

GUP IMPACT ONTO THE ENTROPY BUDGET

- Modification of the Hawking flux when we take into account quantum gravity effects (when the size approaches the Planck length)
  [A. A-S, M. Dabrowski and H. Gohar, PRD 97 (2018) 044029]

- It is possible to directly apply the Generalized uncertainty principle (GUP)

$$\Delta x \Delta p = h[1 + \alpha^2 (\Delta p)^2] \quad \text{where} \quad \alpha = \alpha_0 \frac{l_p}{\hbar}$$

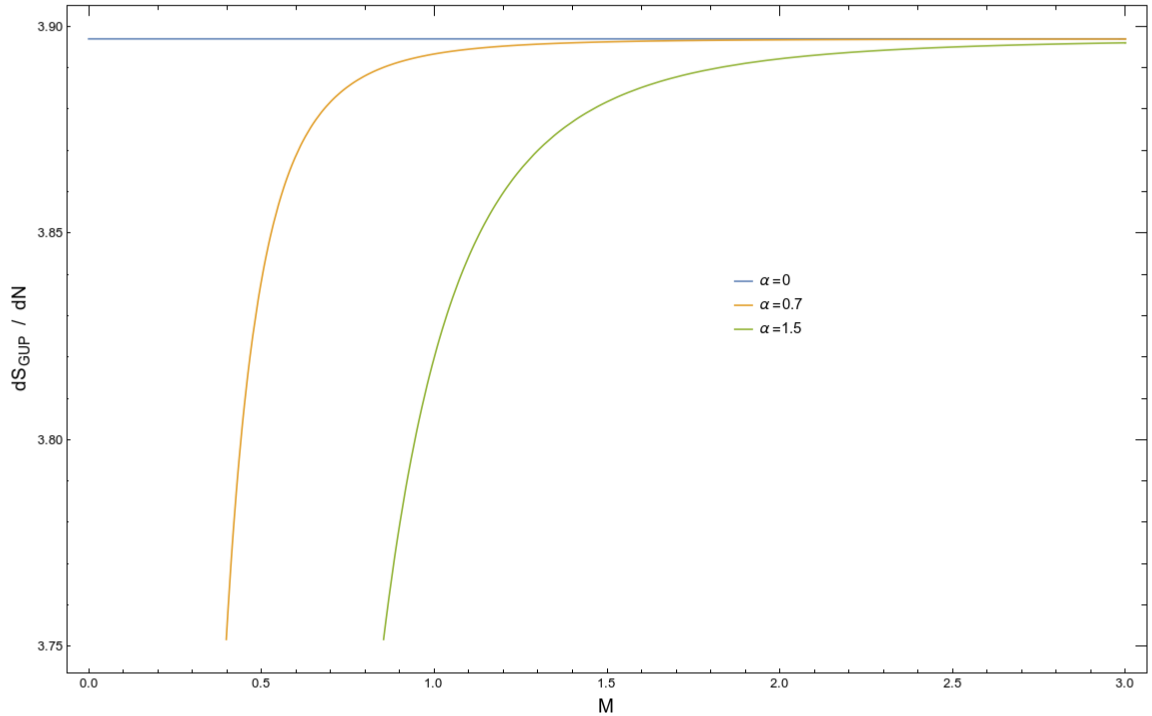
- We obtain a modified Hawking temperature

$$T_{GUP} = T\left[1 + \left(\frac{2\alpha\pi k_B}{c} T\right)^2 + O(\alpha^4)\right]$$

$$\Downarrow$$

$$S_{GUP} = S - \frac{\alpha^2 c^2 m_p^2 k_B \pi}{4} \ln\left(\frac{S}{S_0}\right) + O(\alpha^4)$$

- This leads to a modification of the entropy budget $\frac{dS_{GUP}}{dN} \simeq \frac{k_B \pi^4}{30\zeta(3)}\left[1-\left(\frac{\alpha c}{4}\right)^4\left(\frac{m_p^2}{M}\right)^4\right]$



The entropy budget per emitted particle decreases when the black hole approaches the Planck size

- This leads to a modification of total number of emitted quanta

$$N_{GUP} \simeq \frac{30\zeta(3)}{\pi^4}\left[\frac{4\pi}{m_p^2}M^2 - \frac{\alpha^2 c^2 m_p^2 \pi}{4}\ln\left(\frac{M}{M_0}\right)^2\right]$$

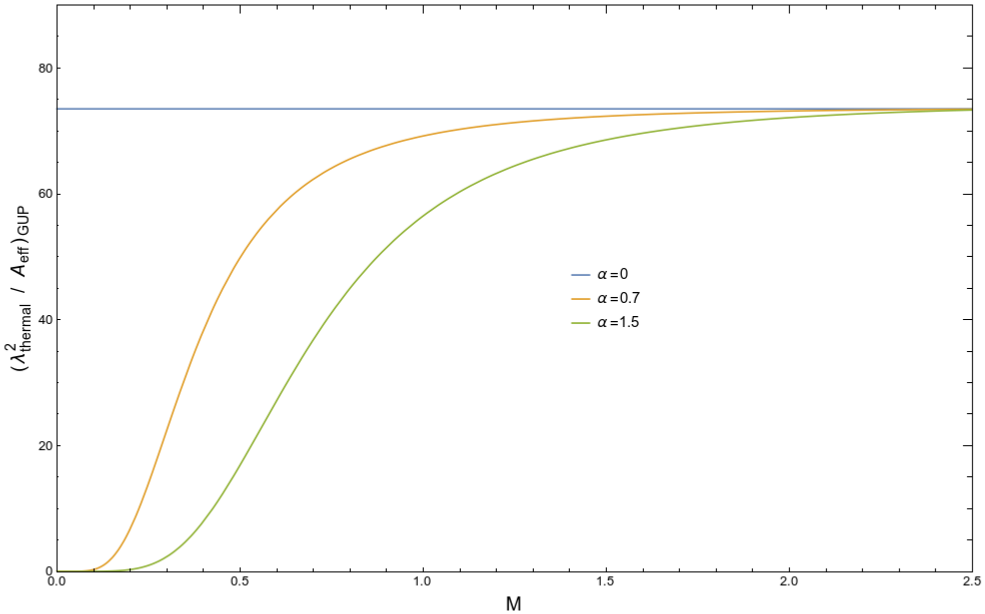
SPARSITY OF HAWKING RADIATION

- Several dimensionless quantities that gave the ratio between an average time between the emission of two consecutive quanta and the natural time scale [F. Gray et al., CQG 33 (2016) 115003]

$$\eta = C \frac{\lambda_{thermal}^2}{gA} \gg 1 \qquad \text{where} \quad \lambda_{thermal} = 2\pi\hbar c/(k_B T)$$

- GUP modification

$$\frac{\lambda_{thermal}^2}{A_{eff}}\Big|_{GUP} = \frac{64\pi^3}{27} \frac{M^6}{\left[M^2 - \left(\frac{\alpha c}{4}\right)^2 m_p^4 \ln\left(\frac{M^2}{M_0^2}\right)\right]\left[M^2 + \left(\frac{\alpha c}{4}\right)^2 m_p^4\right]^2}$$



Hawking flux is no longer sparse

DISCUSSION

- There is no "information puzzle" in burning a lump of coal ⟶ entropy budget = "hidden information" in the correlations
- We have modelled the coarse-graining procedure in a quantifiable and controllable manner ⟶ starting point
- In a black hole system, we calculated the classical thermodynamics entropy and the Bekenstein entropy and they compensate perfectly
- Quantum mechanically, the bipartite Page system give rise to not well-undestood physics ⟶ new tripartite model, which quantum entropy completely agree with the classical expected results (and multi-partite extension)
- When we restrict attention to a particular subsystem we perceive an amount of entanglement entropy (a loss of information)= entropy that is codified in the correlations between the subsystems ⟶ no weird physical effects
- We have investigated quantum gravity modifications to the entropy and temperature of an evaporating black hole expressed by the GUP

Thank you for your attention!

References:

[1] A. A-S, Matt Visser, "*On burning a lump of coal",* Phys. Lett. B 757 (2016) 383.

[2] A. A-S, Matt Visser, "*Entropy budget in black hole evaporation process",* Universe 3 (2017), 58.

[3] A. A-S, Matt Visser, "*Coarse graining Shannon and von Neumann entropies"*, Entropy 2017, 19(5), 207. Special Issue Black Hole Thermodynamics II.

[4] A. A-S, Matt Visser, "*Multi-partite analysis of average-subsystems entropies"*, Phys. Rev. A 96 (2017) no.5, 052302.

[5] A. A-S, Matt Visser, "*Entropy/information flux in Hawking radiation",* Phys. Lett. B 776 (2018) 10-16.

[6] A. A-S, Mariusz Dabrowski, Hussain Gohar, "*Generalized uncertainty principle impact onto the black holes information flux and the sparsity of Hawking radiation"*, Phys. Rev. D 97 (2018) no.4, 044029.

[7] A. A-S, Matt Visser, "*Gravitational collapse: The big coarse graining"*, work in progress.