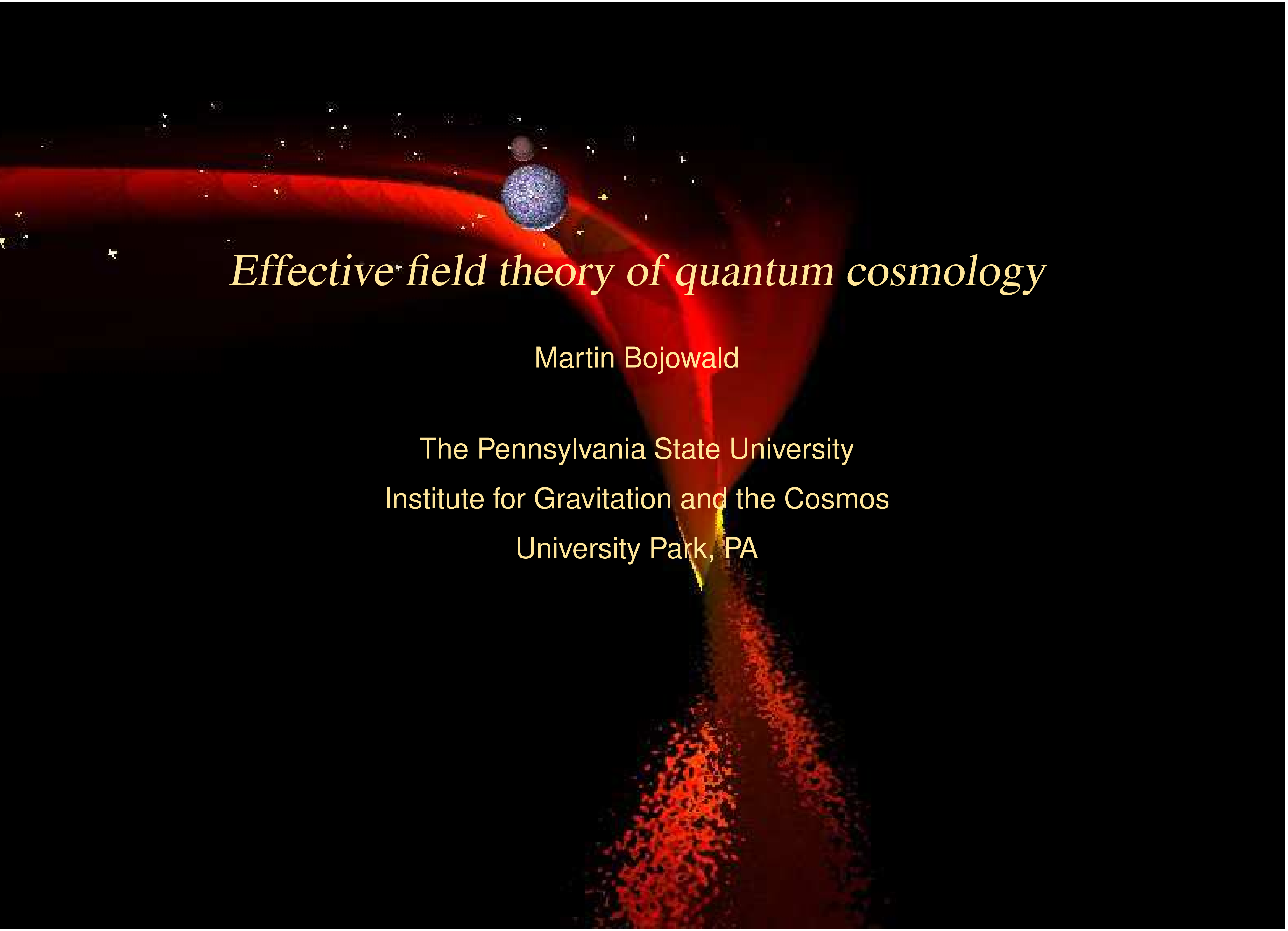

Effective field theory of quantum cosmology

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Quantum gravity conundrum

Interacting quantum theory of many degrees of freedom.

Need approximations and assumptions.
Hard to find good ones without observational assistance.

Still, need phenomenology to suggest promising experiments.

General approach: Effective field theory.
Parameterize large class of potential outcomes.

Main ingredients for quantum cosmology:

- → Minisuperspace approximation.
- → BKL scenario.
- → Infrared regularization.

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Model minisuperspace model

[with Brahma: arXiv:1509.00640]

Lagrangian $L=\int \mathrm{d}^3x \left(\frac{1}{2}\dot{\phi}^2-\frac{1}{2}|\nabla\phi|^2-W(\phi)\right)$ reduced to

$$L_{\rm mini}=V_0\left(\frac{1}{2}\dot{\phi}^2-W(\phi)\right)$$

with $V_0=\int \mathrm{d}^3x$, ϕ spatially constant.

Momentum $p=V_0\dot{\phi}$, Hamiltonian $H_{\rm mini}=\frac{1}{2}p^2/V_0+V_0W(\phi)$.

Quantized to

$$\hat{H}_{\rm mini}=\frac{1}{2}\frac{\hat{p}^2}{V_0}+V_0W(\hat{\phi})$$

Quantum corrections depend on V_0:

Quantum cosmology is a quantum theory of regions (patches), not of the metric or scale factor.

Effective potentials

Minisuperspace:

$$W_{\mathrm{eff}}^{\mathrm{mini}}(\phi) = W(\phi) + \frac{1}{2V_0}\hbar\sqrt{W''(\phi)}$$

Coleman–Weinberg potential: [Coleman, Weinberg: PRD 7 (1973) 1888]

$$W_{\mathrm{eff}}(\phi) = W(\phi) + \frac{1}{2}\hbar\int \frac{\mathrm{d}^4k}{(2\pi)^4}\log\left(1 + \frac{W''(\phi)}{||\mathbf{k}||^2}\right)$$

After k^0-integration (or canonical derivation):

$$W_{\mathrm{eff}}(\phi) = W(\phi) + \frac{1}{2}\hbar\int \frac{\mathrm{d}^3k}{(2\pi)^3}\left(\sqrt{|\vec{k}|^2 + W''(\phi)} - |\vec{k}|\right)$$

[with Brahma: arXiv:1411.3636]

Comparison

Minisuperspace potential

$$W(\phi) + \frac{1}{2V_0}\hbar\sqrt{W''(\phi)}$$

as infrared-contribution from quantum field theory:

$$\frac{1}{2}\hbar\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left(\sqrt{|\vec{k}|^2 + W''(\phi)} - |\vec{k}|\right)$$

integrated over $|\vec{k}| \leq k_{\max} = 2\pi/V_0^{1/3}$,

$$W_{\mathrm{eff}}(\phi) \approx W(\phi) + \frac{\hbar}{12\pi^2}k_{\max}^3\sqrt{W''(\phi)} = W(\phi) + \frac{2\pi}{3V_0}\hbar\sqrt{W''(\phi)}$$

Infrared gravity

Complicated infrared behavior from massless excitations:

$\sqrt{|\vec{k}|^2 + W''(\phi)} \approx |\vec{k}| + \frac{1}{2}W''(\phi)/|\vec{k}|$ if $W''(\phi) \ll |\vec{k}|^2$.

Agrees with detailed analysis for gravity. [Wetterich: arXiv:1802.05947]

Quantum cosmology: Parameterize $Q \propto (V_0^{1/3}a)^{2(1-x)}$, $P \propto (V_0^{1/3}a)^{2x}V_0^{1/3}\dot{a}$, $(\Delta Q)^2 \propto (V_0^{1/3}a)^{4y}$, constant x and y.

Uncertainty relation $(\Delta Q)^2(\Delta P)^2 \geq \hbar^2/4$: $(\Delta P)^2 \propto V_0^{-4y/3}$.

Therefore,

$$V_0\frac{(\Delta Q)^2}{Q^2} \propto k^{1-4(x+y)} \quad , \quad V_0\frac{(\Delta P)^2}{P^2} \propto k^{4(x+y)-1}$$

implies inverse-k terms in Hamiltonian (unless $x+y=1/4$).

Minisuperspace approximation

Minisuperspace $\hbar$-corrections approximate interaction of QFT modes with wavelengths greater than averaging volume V_0:

$\longrightarrow$ Includes modes with $|\vec{k}| \leq k_{\max} = 2\pi/V_0^{1/3}$.

$\longrightarrow$ Replaces mode integral by integrand at $|\vec{k}| = 0$ times k-volume.

Acceptable approximation in regimes with
(i) *inhomogeneity on scales greater than* V_0 if (ii) V_0 *large.*

Both conditions fulfilled in late-universe cosmology.

Approach to a spacelike singularity

Inhomogeneity grows in comoving region (constant V_0).

$\longrightarrow$ Should shrink V_0 to maintain minisuperspace description: Infrared renormalization. Expect mixed states.

No unitary transformation to change V_0.
Cannot use single Hilbert space. Effective theory required.

$\longrightarrow$ BKL: Can assume homogeneous geometry right up to spacelike singularity, but no lower limit placed on V_0.

Small V_0: Local Bianchi IX geometry, but not a Bianchi IX model.

$\longrightarrow$ Minisuperspace approximation less reliable near spacelike singularities.

Bohmian quantum cosmology

[Pinto-Neto, Struyve, arXiv:1801.03353]

Hamiltonian

$$H = \frac{1}{2} f^{ab}(q) p_a p_b + U(q)$$

implies quantum potential

$$W_{\mathrm{Q}} = -\frac{1}{2\sqrt{f}|\psi|} \frac{\partial}{\partial q^a} \left(f^{ab} \sqrt{f} \frac{\partial}{\partial q^b} |\psi| \right)$$

If $f^{ab} \propto V_0^{-2}$, $W_{\mathrm{Q}} \propto V_0^{-2}$ enhanced near spacelike singularity with small V_0.

Bounce models more secure in effective picture.

[e.g. Falciano, Pinto-Neto, Santini: arXiv:0707.1088]

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Affine quantization

[Bergeron, Czuchry, Gazeau, Malkiewicz, Piechocki, arXiv:1501.07871]

Effective Friedmann equation

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} + \frac{k_2}{\ell^2 a^6} = \frac{8\pi G}{3}\rho(a)$$

with effective energy density $\rho(a) = k_3\hbar(N+1)/V_0 a^4$ from harmonic anisotropies, and $\ell = V_0/\ell_{\rm P}^2$.

$\longrightarrow$ Behavior of $\rho(a)$ agrees with $\hbar\sqrt{W''}/V_0$.

$\longrightarrow$ Repulsive $\ell^{-2}a^{-6} \propto \hbar^2/V_0^2$, responsible for bounce.

$\longrightarrow$ Bounce more secure for small V_0, but: Do higher-order $(\hbar/V_0)$-corrections in $\rho(a)$ compete with repulsive term?

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Loop quantum cosmology

Effective Friedmann equation [Vandersloot, gr-qc/0502082]

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\rm QG}}\right)$$

with

$$\rho_{\rm QG} = \frac{3}{8\pi G\delta^2 (V_0^{1/3}a)^{2(1+2x)}}$$

Analysis usually done for macroscopic V_0.

[Ashtekar, Pawlowski, Singh, gr-qc/0604013, . . .]

For $x > -1/2$, $\rho/\rho_{\rm QG} \propto V_0^{2(1+2x)/3}$ decreases for small V_0.
Quantum effects weaker.

Borderline case $x = -1/2$ independent of V_0, but sensitive to quantum fluctuations. Non-bouncing solutions do exist.

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Summary

Effective field theory of quantum cosmology combines BKL scenario with effects from quantum field theory.

$\longrightarrow$ Shows qualitative differences between various approaches.

$\longrightarrow$ Strengthens quantum effects based on fluctuations.

$\longrightarrow$ Reveals spurious effects in loop quantum cosmology, where large V_0 is often assumed even for early-universe models.

$\longrightarrow$ Highlights main problem of minisuperspaces:

How to reconcile small regions in asymptotic regime of BKL with infrared truncation of quantum field theory.