Unruh effect without spacetime

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Puzzles of BH thermodynamics

- **Bekenstein*** suggested that BH have entropy

$$S \sim \frac{A}{l_{Pl}^2}$$

- Spectacularly confirmed by Hawking** BH radiate at temperature

$$T_H = \frac{1}{8\pi GM}$$

* Phys. Rev. D7, 2333 (1973); ** Nature 248, 30 (1974):

Puzzles of BH thermodynamics

- To date at least two major issues remain puzzling:
  - The enigmatic nature of the degrees of freedom that BH entropy is counting;
  - The fate of unitarity in BH quantum evaporation: do BHs evolve pure states into mixed states?

Puzzles of BH thermodynamics

- To date at least two major issues remain puzzling:
  - The enigmatic nature of the degrees of freedom that BH entropy is counting;
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Crucial to the BH thermodynamic puzzles is the quantum temperature perceived by accelerated observer, but not by inertial one.

Horizon temperature

- The inequivalence of the notion of particle for Rindler and Minkowski observers was first discussed by Fulling 45 years ago

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Nonuniqueness of Canonical Field Quantization in Riemannian Space-Time*

Stephen A. Fulling†
University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201
(Received 3 November 1972)

Horizon temperature

- Unruh* motivated by Hawking discovery that black holes radiate thermally at T_H, associates temperature to the Rindler horizon

$$T_U = \frac{a}{2\pi}$$

- Universal property of all causal horizons (Jacobson and Parentani**)

* Phys. Rev. D 14, 870 (1976), ** Found. Phys. 33, 323 (2003))

- In this talk I will describe the simplest system exhibiting a **connection between a geometric notion of boundary and thermal quantum states.**
- Minimal setting: only **group theoretic ingredients** associated with symmetries of space-time
  - No quantum fields
  - No space-time
  - No metric

Accelerated observers

- Accelerated worldline is the Lorentz orbit of the vector (0; 1/α)

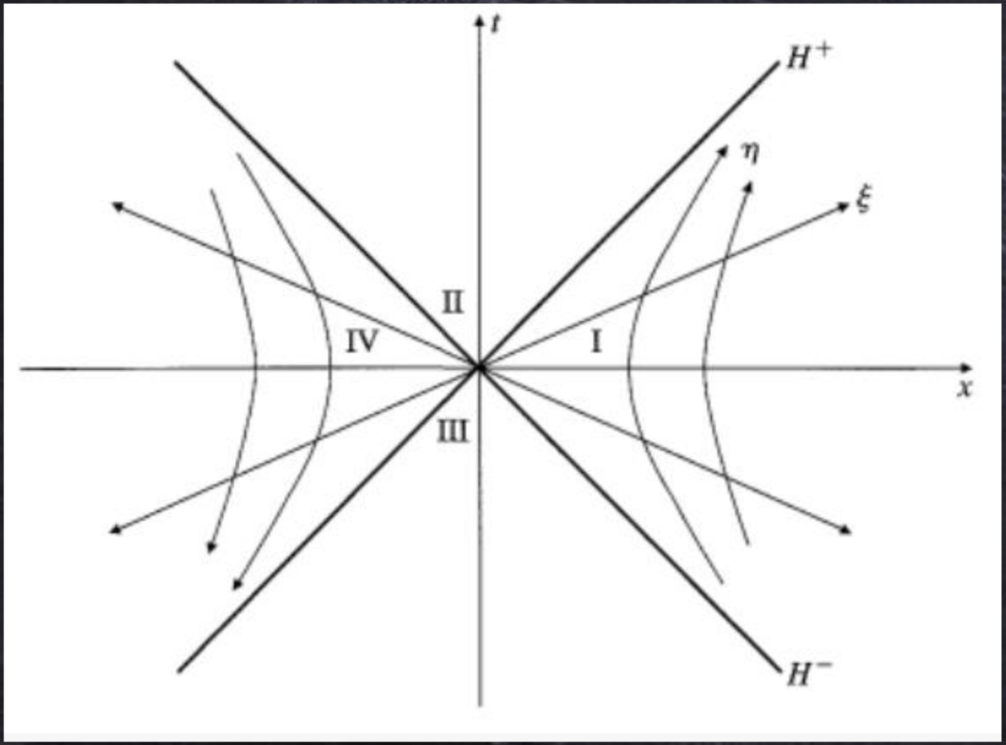
$$t^2(\tau) - x^2(\tau) = \frac{1}{\alpha^2}$$

- Lorentz transformations move points along the orbit of constant acceleration; but what is the transformation that changes the acceleration?
- It is the dilation that does the job.

$$(t,x) \to (t',x') = e^{\delta}(t,x)$$

Rindler space

- We define the Rindler coordinates with the help of the boost and dilation parameters.



$$t = \frac{1}{a} e^{a\xi} \sinh a\eta$$

$$x = \frac{1}{a} e^{a\xi} \cosh a\eta$$

Weyl–Poincaré algebra

- We have two translation operators P_t and P_x generating translation in Minkowski space;
- We have two translation operators N and D generating translation in Rindler space (a=1).

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- We have two translation operators N and D generating translation in Rindler space (a=1).

$$[P_t, P_x] = 0, \; [D, N] = 0$$

$$[N, P_t] = iP_x, \; [N, P_x] = iP_t$$

Weyl-Poincaré algebra

- Important reminder:

The eigenfunctions of translational symmetry generators with positive eigenvalue of the time-like one have the interpretation of one-particle states.

Weyl-Poincaré algebra

- We can form light-cone generators (a=1)

$$P_{\pm} \equiv P_t \pm P_x\,, \quad R_{\pm} \equiv -\frac{1}{2}(N \pm D)$$

- Which satisfy a simple algebra

$$[P_{\pm}, R_{\pm}] = \pm i P_{\pm}$$

$$[P_{\pm}, R_{\mp}] = 0$$

Comments

- These are two copies of the ax +b algebra which generate affine transformations of the real line with (-)+ sign: preserves (anti-)orientation.

$$[P_{\pm}, R_{\pm}] = \pm i P_{\pm}$$

$$[P_{\pm}, R_{\mp}] = 0$$

Comments

- We concentrate on the algebra

$$[P,R]=iP$$

- which is the simplest non-trivial Lie algebra, with P, R having a geometric interpretation of translation and dilation of the real line.
- What's more important, P is translation of the line, while R is translation of half-line. This is a generic feature of systems with (null) boundaries.

P-particle states

- Representations of the P generator

$$P|\pm k\rangle = \pm k|\pm k\rangle, \quad k \in \mathbb{R}^+$$

- Hilbert space inner product

$$\langle\psi|\psi'\rangle = \int_0^\infty \frac{dk}{k}\,\bar{\psi}(k)\psi'(k) = \int_0^\infty \frac{dk}{k}\,\langle\psi|k\rangle\langle k|\psi'\rangle$$

$$\int_0^\infty \frac{dk}{k}|k\rangle\langle k| = 1, \qquad \langle k|k'\rangle = k\,\delta(k-k')$$

- We call $|k\rangle$ a P-particle state

Plane waves: P-state

- Coordinate Hilbert space $|x\rangle$ irreps of the **abelian subgroup generated** by P

$$\langle x\,|\,k\rangle = \frac{1}{\sqrt{2\pi}} e^{-ikx}, \quad \int_{-\infty}^{\infty} dx\,|\,x\rangle\langle x\,| = 1, \; \langle x\,|\,x'\rangle = \delta(x - x')$$

$$P = i\frac{d}{dx}, \quad R = ix\frac{d}{dx}, \quad x \in \mathbb{R}$$

- $|k\rangle$ is a P-particle state

Plane waves: R-state

- Coordinate Hilbert space $|\xi\rangle$ irreps of the **abelian subgroup generated by** R

$$\langle \xi | \omega \rangle = \frac{1}{\sqrt{2\pi}} e^{-i\omega\xi}, \quad \langle \omega | \omega' \rangle = \omega \delta(\omega - \omega'), \ \langle \xi | \xi' \rangle = \delta(\xi - \xi')$$

$$R = i\frac{d}{d\xi}, \quad P = ie^{-\xi}\frac{d}{d\xi}, \qquad \xi \in \mathbb{R}$$

- $|\omega\rangle$ is an R-particle state

Connecting P and R states

- What is the relation between x and ξ representations?

$$R = i\frac{d}{d\xi} = ix\frac{d}{dx} \Rightarrow \frac{dx}{d\xi} = x\,,$$

$$x = \pm e^{\xi}$$

$$P = i\frac{d}{dx} = ie^{-\xi}\frac{d}{d\xi} \Rightarrow \frac{d\xi}{dx} = e^{-\xi}$$

The space coordinate ξ of the R-representation covers either the positive or the negative half of the spectrum of the space coordinate x of the P-representation.

Connecting P and R states

- We focus on $x = e^{\xi}\,,\ \ \xi = \log x$
- There is an isomorphism

$$|\xi\rangle \simeq |x = e^{\xi}\rangle \equiv |x\rangle_+$$

- We have the following maps between coordinate space wavefunctions

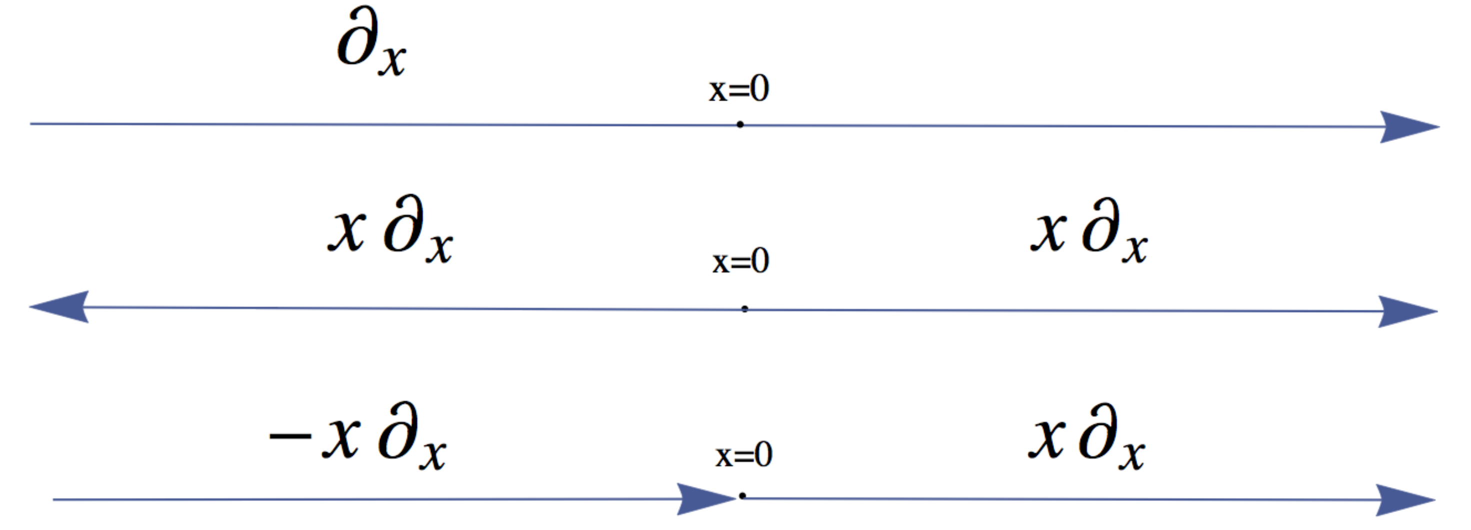
$$|\omega\rangle = \frac{1}{\sqrt{2\pi}} e^{-i\omega\xi} \Leftrightarrow {}_+\langle x|\omega\rangle = \frac{1}{\sqrt{2\pi}} x^{-i\omega}$$

$$\langle x|k\rangle = \frac{1}{\sqrt{2\pi}} e^{-ikx} \Leftrightarrow \langle \xi|k\rangle_+ = \frac{1}{\sqrt{2\pi}} e^{-ike^{\xi}}$$

Extending R to the line

- We would like to define an additional copy of the ξ-representation **isomorphic** to x-states with support on the negative half line.
- Subtle point: the generator P has a global right moving orientation; R is right moving on the positive half line and left moving on the negative.

Extending R to the line



- To cover the negative x spectrum with right moving R-wavefunctions we need to introduce $|\bar{\xi}\rangle$ states carrying irreps of

$$\bar{R} = -R = -ix\frac{d}{dx} = i\frac{d}{d\xi}$$

Extending R to the line

- Now we have an isomorphism

$$|\bar{\xi}\rangle \simeq |x = -e^{-\xi}\rangle \equiv |x\rangle_-$$

- so that ξ belonging to (-∞, ∞) covers the range (-∞, 0) of x.
- We have the wavefunctions

$$\langle\bar{\xi}|\omega\rangle = \frac{1}{\sqrt{2\pi}}e^{-i\omega\xi} \Leftrightarrow {}_-\langle x|\omega\rangle = \frac{1}{\sqrt{2\pi}}(-x)^{i\omega}$$

- and $$\langle x|k\rangle = \frac{1}{\sqrt{2\pi}}e^{-ikx} \Leftrightarrow \langle\bar{\xi}|k\rangle_- = \frac{1}{\sqrt{2\pi}}e^{ike^{-\xi}}$$

Summary (so far)

$$|x\rangle$$

$$|k\rangle = \begin{cases} |k\rangle_+ & \text{(positive energy)} \\ |k\rangle_- & \text{(negative energy)} \end{cases}$$

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$|x\rangle_-$ $\qquad\qquad$ $|x\rangle_+$

$|\bar{\xi}\rangle$ $\qquad\qquad$ $|\xi\rangle$

$$|\omega\rangle = \begin{cases} |\omega\rangle_+ & \text{(positive energy)} \\ |\omega\rangle_- & \text{(negative energy)} \end{cases}$$

Decomposition of identity

- To define a R-state on the whole real line we write the resolution of identity

$$\int_{-\infty}^{\infty} dx|x\rangle\langle x| = \int_0^{\infty}\frac{dx}{x}|x\rangle_{++}\langle x| + \int_{-\infty}^{0}\frac{dx}{x}|x\rangle_{--}\langle x| = 1$$

$${}_{\pm}\langle x|x'\rangle_{\pm} = \pm x\,\delta(x-x')$$

- From

$${}_{+}\langle x|\omega\rangle = \frac{1}{\sqrt{2\pi}}x^{-i\omega},\quad {}_{-}\langle x|\omega\rangle = \frac{1}{\sqrt{2\pi}}(-x)^{i\omega}$$

Normalization:

$$\int_0^{\infty}\frac{dx}{x}x^{-i(\omega-\omega')} = 2\pi\,\delta(\omega-\omega')$$

$${}_{\mathbb{R}}\langle\omega|\omega'\rangle_{\mathbb{R}} = 2\omega\,\delta(\omega-\omega')$$

$$|\omega\rangle_{\mathbb{R}} = \sqrt{\frac{\omega}{2\pi}}\left(\int_0^{\infty}\frac{dx}{x}x^{-i\omega}|x\rangle_{+} - \int_0^{\infty}\frac{dx}{x}x^{i\omega}|-x\rangle_{-}\right)$$

Final step

- Calculate the overlap between k and states

$$ {}_{\pm}\langle k \,|\, \omega \rangle_{\mathbb{R}} = \frac{\sqrt{\omega}}{2\pi} \int_0^{\infty} \frac{dx}{x} x^{\mp i\omega} e^{ikx} = \pm \frac{\sqrt{\omega}}{2\pi} k^{\pm i\omega} e^{\mp\frac{\pi\omega}{2}} \Gamma(\mp i\omega) $$

- How many P-particles are there in the state ω?

$$ \int_0^{\infty} \frac{dk}{k} {}_{\mathbb{R}}\langle \omega \,|\, k \rangle_{++} \langle k \,|\, \omega' \rangle_{\mathbb{R}} = \frac{\delta(\omega - \omega')}{e^{2\pi\omega} - 1} $$

Thus …

The state ω carries a
thermal distribution of P-particles at temperature T=1/2π.
(reinstating the acceleration parameter a this gives exactly the Unruh temperature T=a/2π.)

Summary

- I described the simplest system exhibiting a connection between a geometric notion of boundary and thermal quantum states.
- Such relation is at the basis of the thermal behavior of quantum systems in the presence of black holes and for accelerated observers: crucial role in quantum gravity.

What next?

- The $ax + b$ group plays a key role in a variety of contexts: non-commutative geometry, affine quantization, quantum cosmology, …
- Could the effect described in this talk be relevant for these applications?

What next?

- The setup used here in the case of 1D system looks pretty generic and the role played by Weyl-Poincaré algebra seems to be essential also in the case of higher dimensional systems with null boundaries.