Multiple possible resolutions of the cosmological singularity in quantum gravity

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Albert Einstein Institute - Potsdam-Golm

2nd workshop on “Singularities of general relativity and their quantum fate”
Banach Center
Warsaw, Poland, EU
22/05/2018

General perspective and plan

General idea:

Space(time) as a (background independent) quantum many-body system

Continuum spacetime and geometry as emergent notions from collective quantum properties

Formalism used: Group Field Theory

- Building blocks from quantized discrete geometry in algebraic language
- Recast in QFT formalism in order to use "standard tools"
- Incorporate insights and results from related approaches
- Study collective quantum behaviour and extract effective continuum physics

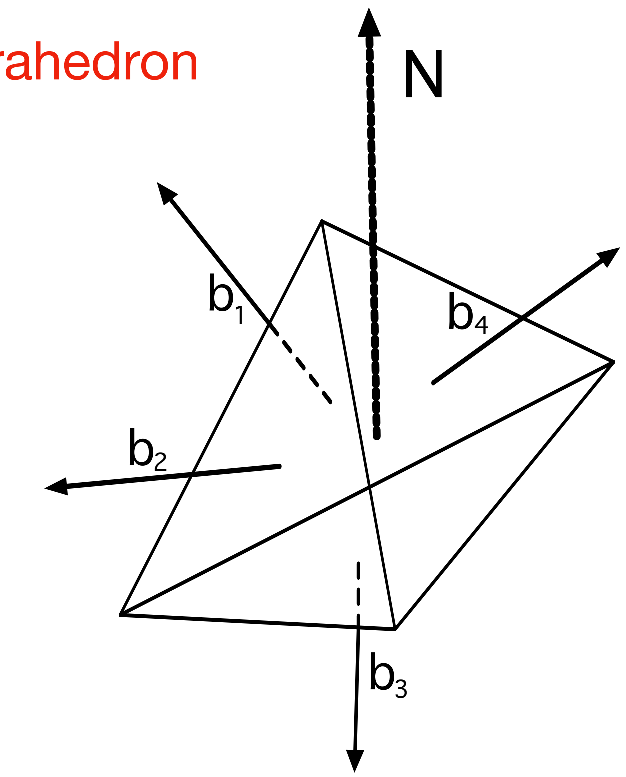
Plan:

- Introduction to GFT: building blocks, dynamics, relation with other formalisms
- GFT condensate cosmology: idea, basic results
- Possible fate(s) of cosmological singularity in quantum gravity

An “atom of space”

Barbieri ’97; Baez, Barrett, ’99; Rovelli, Speziale, ’06; Bianchi, Dona, Speziale, ’10; ……

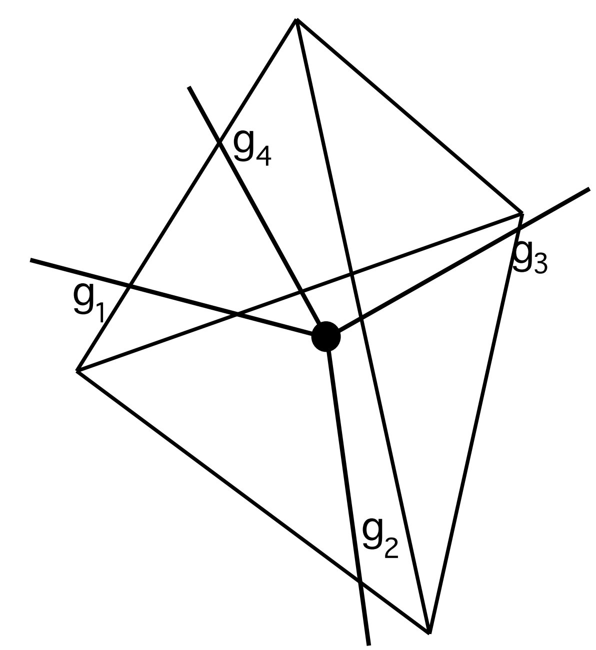
Elementary building block of 3d space: single polyhedron - simplest example: a tetrahedron



Classical geometry in group-theoretic variables

4 vectors normal to triangles that close (lying in hypersurface with normal N)

$$\boxed{A_i\, n_i^I \;=\; b_i^I \in \mathbb{R}^{3,1} \qquad b_i \cdot N \;=\; 0 \qquad \sum_i b_i \;=\; 0}$$



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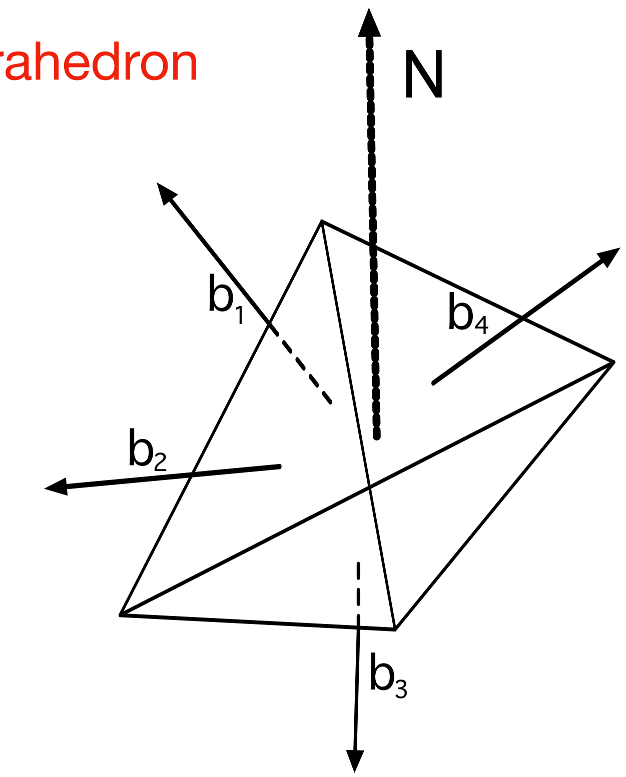

equivalently: constrained 4d area 2-forms:

$$\left(B_i^{IJ} \in \wedge^2\mathbb{R}^{3,1} \simeq \mathfrak{so}(3,1)\,,\, N^I \in \mathcal{T}\mathbb{R}^{3,1}\right) \qquad N_I\,(*B_i^{IJ}) = 0 \qquad \sum_i B_i^{IJ} = 0$$

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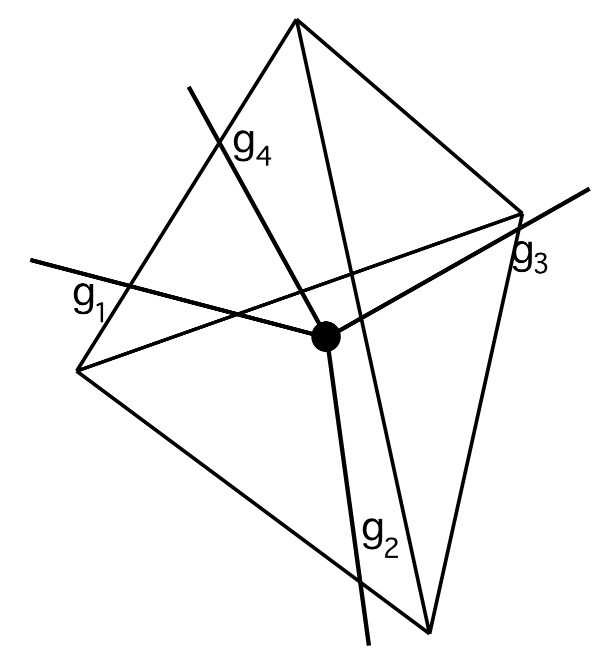

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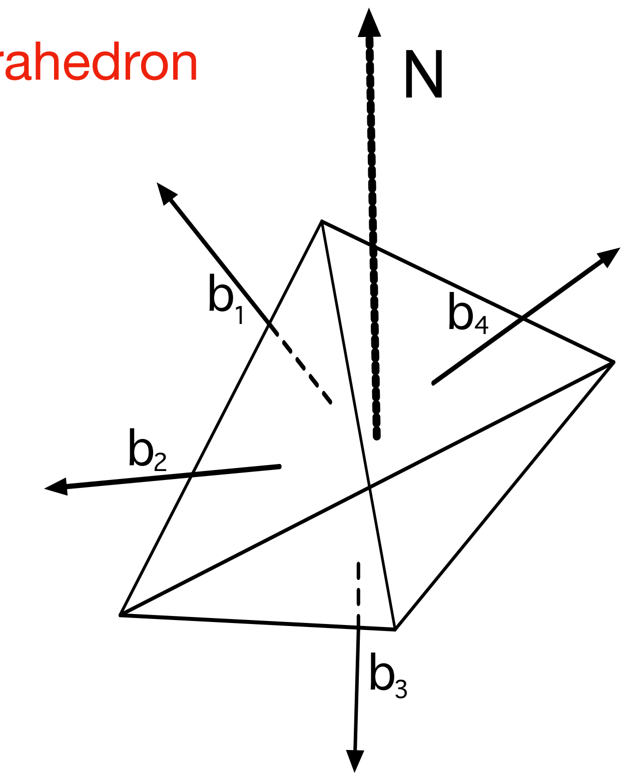

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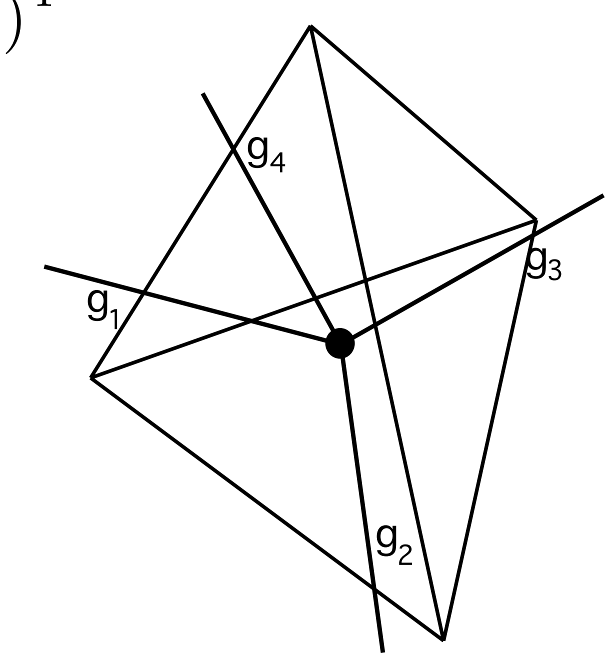
phase space:

$$(\mathcal{T}^* SO(3,1))^4 \simeq (\mathfrak{so}(3,1) \times SO(3,1))^4 \supset (\mathfrak{so}(3) \times SO(3))^4 \simeq (\mathcal{T}^* SO(3))^4$$



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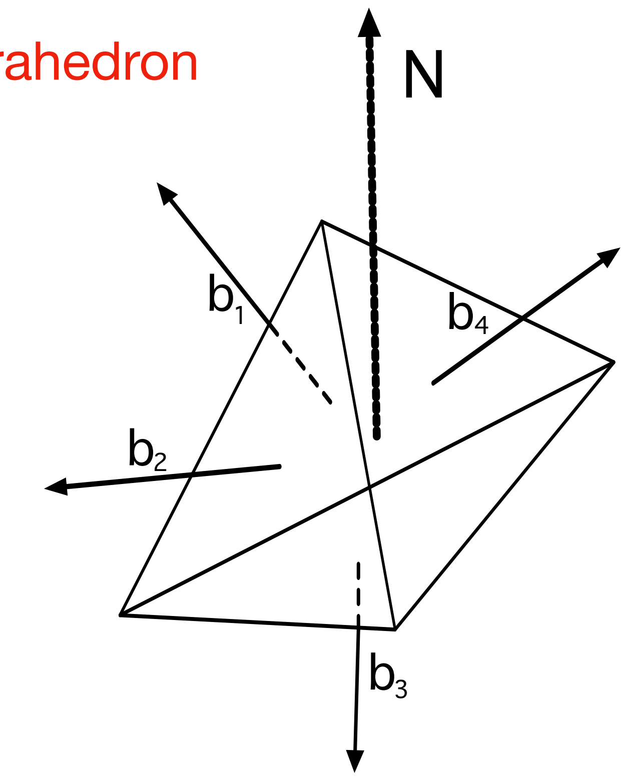

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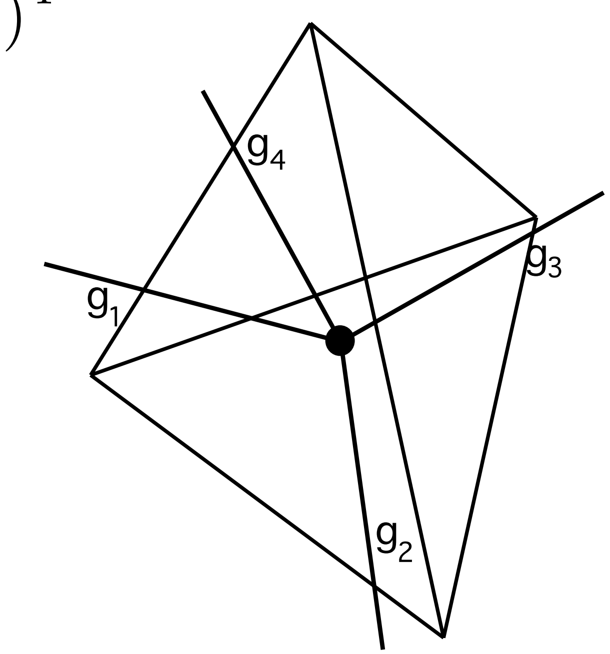

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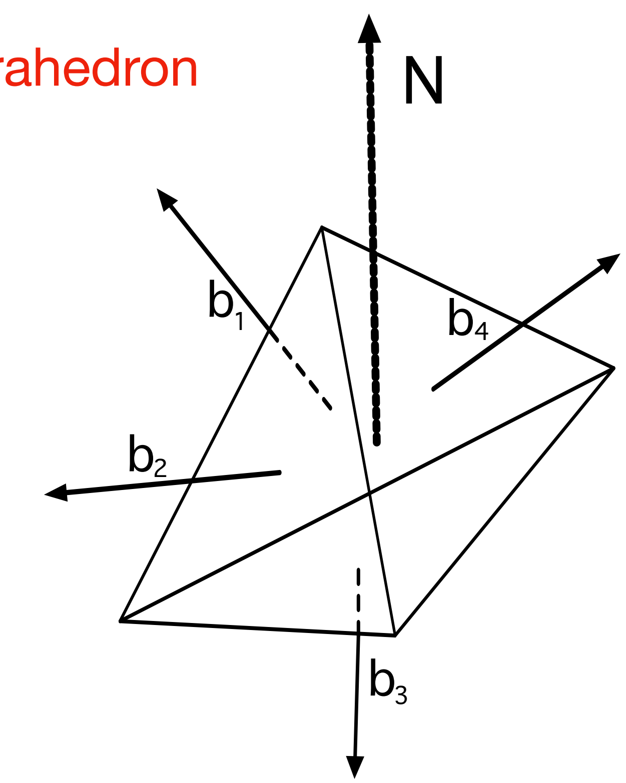

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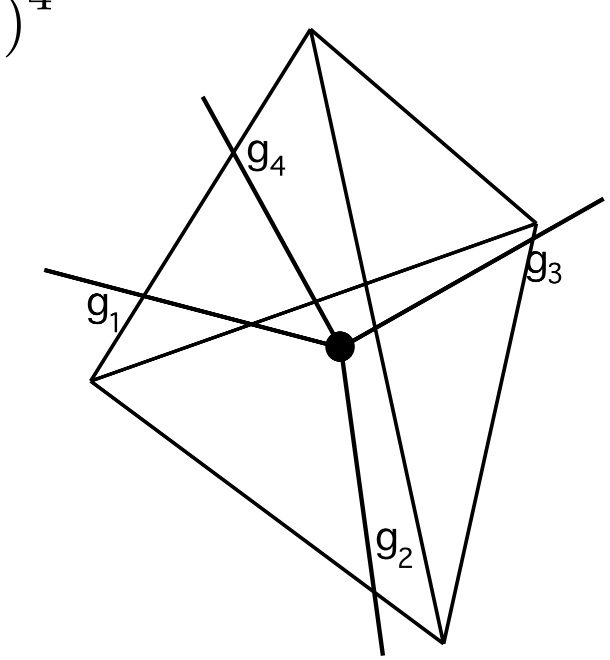
general: $(\mathcal{T}^* G)^{\times d} \simeq (\mathfrak{g} \times G)^{\times d}$



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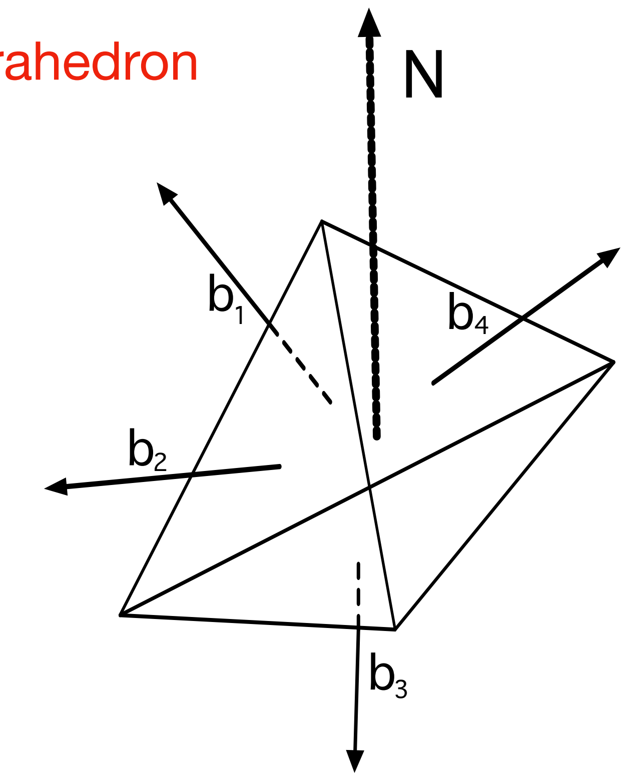

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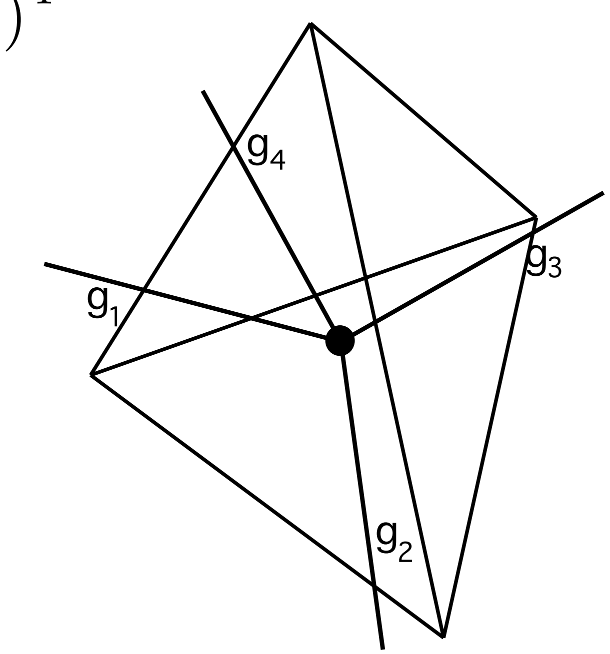
Quantum geometry in group-theoretic variables



Hilbert space

$$\mathcal{H}_v \;=\; L^2\left(G^d; d\mu_{Haar}\right)$$

+ constraints on states

$$B_i^{IJ} \;\rightarrow\; \hat{J}^{IJ} \in \mathfrak{so}(3,1) \qquad b_i^J \;\rightarrow\; \hat{J}_N^i \in \mathfrak{su}(2)$$

spin network vertex

Quantum space as a many-body system

DO, '13

Many-body Hilbert space for "quantum space": Fock space

$$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym\left\{\left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)}\right)\right\}$$

Fock vacuum: "no-space" state $|0>$

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Second-quantised representation: ladder and geometric operators

$$\left[\hat{\varphi}(\vec{g})\,,\,\hat{\varphi}^\dagger(\vec{g}')\right] = \mathbb{I}_G(\vec{g},\vec{g}') \qquad [\hat{\varphi}(\vec{g})\,,\,\hat{\varphi}(\vec{g}')] = \left[\hat{\varphi}^\dagger(\vec{g})\,,\,\hat{\varphi}^\dagger(\vec{g}')\right] = 0$$

$$\rightarrow \quad \widehat{\mathcal{O}_{n,m}}\left(\hat{\varphi},\hat{\varphi}^\dagger\right) = \int [d\vec{g}_i][d\vec{g}'_j]\;\widehat{\varphi}^\dagger(\vec{g}_1)..\widehat{\varphi}^\dagger(\vec{g}_m)\mathcal{O}_{n,m}\left(\vec{g}_1,..,\vec{g}_m,\vec{g}'_1,..,\vec{g}'_n\right)\widehat{\varphi}(\vec{g}'_1)..\widehat{\varphi}(\vec{g}'_n)$$

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e.g. total space volume (extensive quantity):

$$\hat{V}_{tot} = \int [dg_i][dg'_j]\hat{\varphi}^{\dagger}(g_i)\,V(g_i,g'_j)\,\hat{\varphi}(g'_j) = \sum_{J_i} \hat{\varphi}^{\dagger}(J_i)\,V(J_i)\,\hat{\varphi}(J_j)$$

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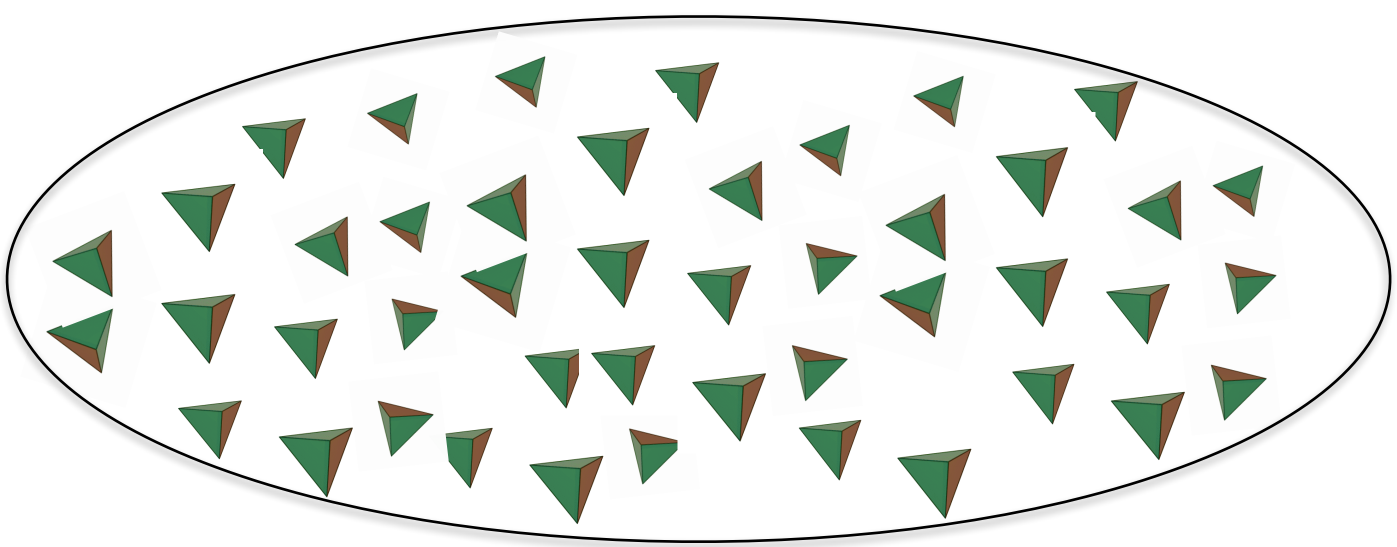
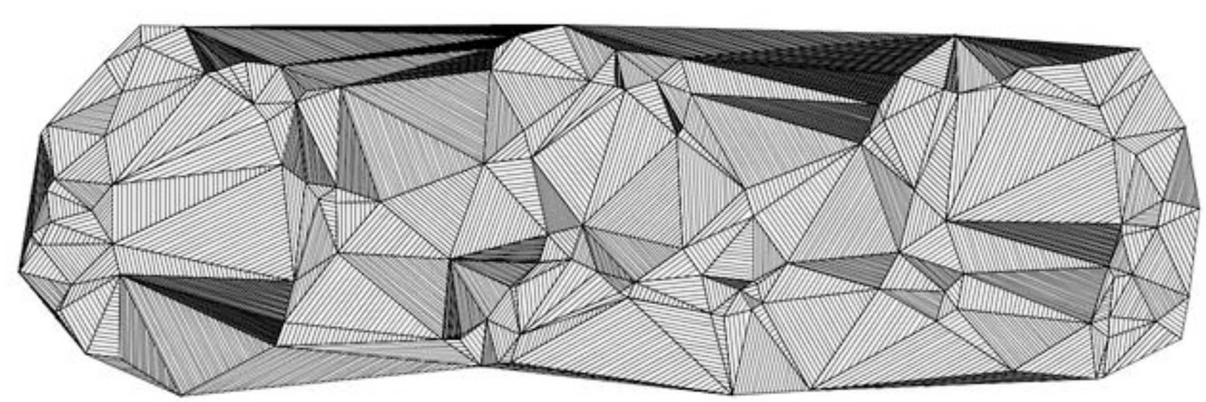
Quantum space as an system of many quantum polyhedra/spin network vertices

generic states not very "spacey" at all - "connected" many-body states a little more "spacey"



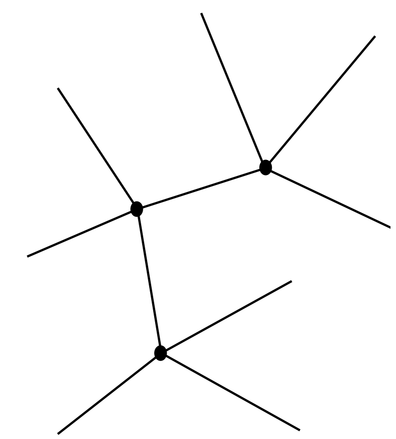


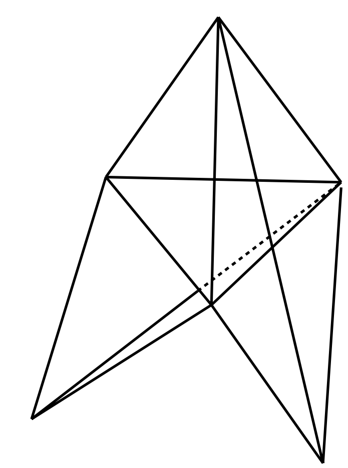
Quantum space as a many-body system

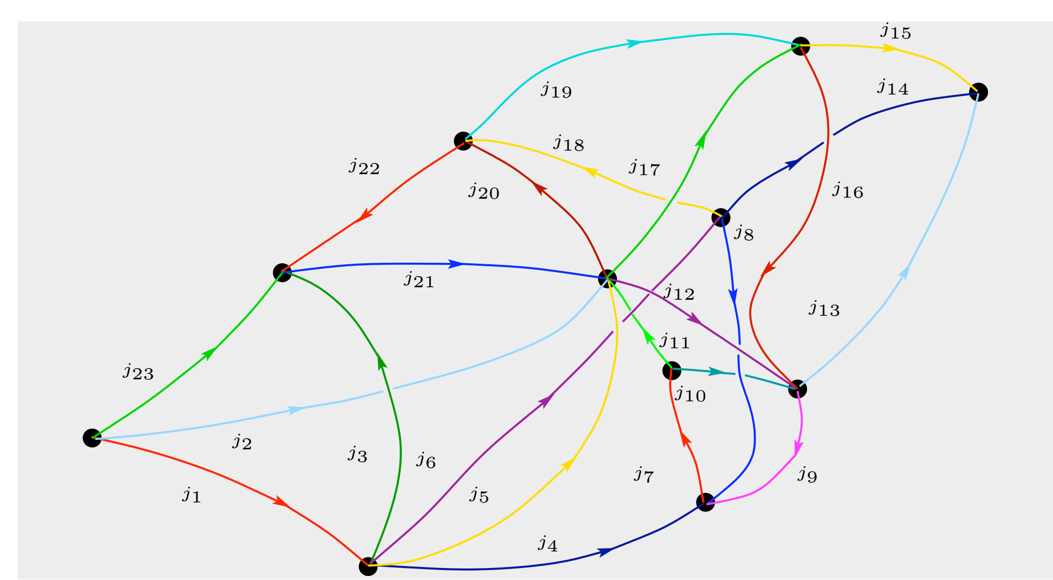
DO, ‘13

Forming extended structures: gluing building blocks — —-> states on connected graphs/simplicial complexes

$$\mathcal{H}_\gamma \subset \mathcal{H}_V \qquad \Psi\gamma(G_{ij}^{ab}) = \prod_{[(ia),(jb)]} \int_G d\alpha_{ij}^{ab}\, \phi_V(\ldots, g_{ia}\, \alpha_{ij}^{ab}, \ldots, g_{jb}\alpha_{ij}^{ab}, \ldots) = \Psi\gamma(g_{ia}(g_{jb})^{-1})$$







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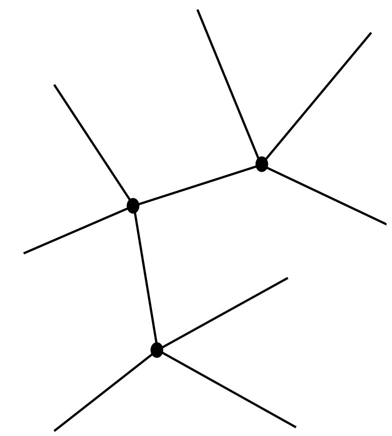
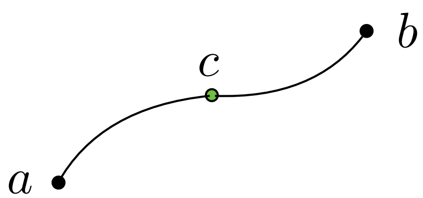
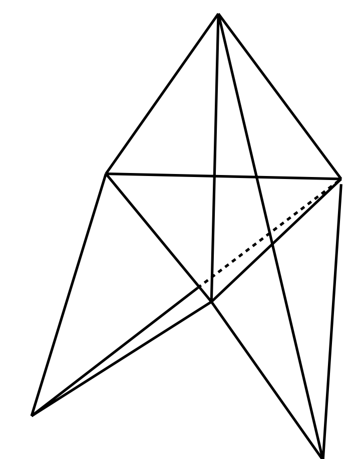


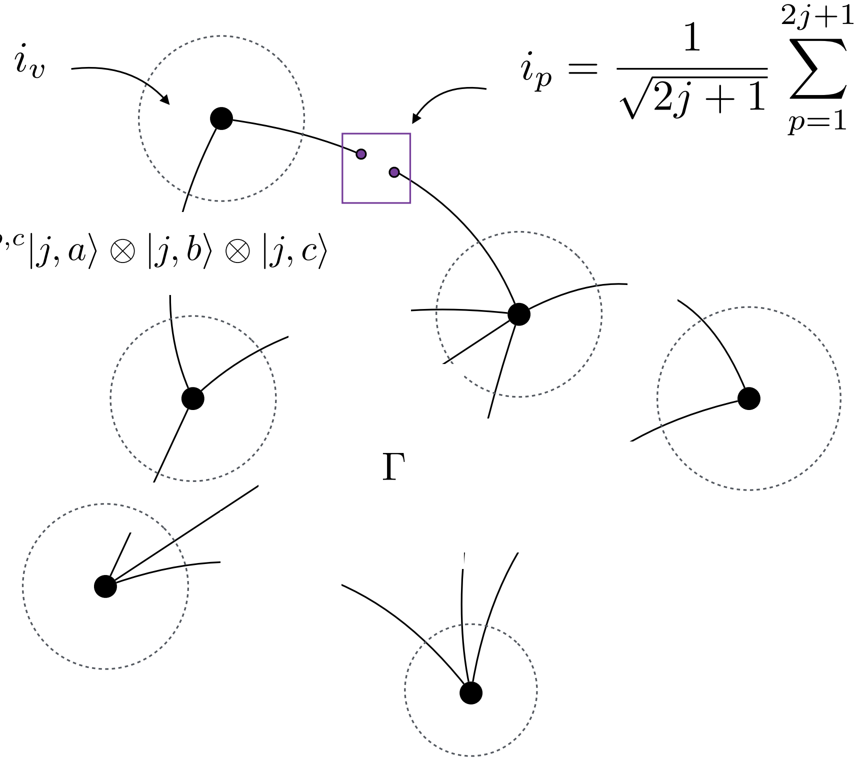
Gluing = connectivity = entanglement between "atoms of space"



$$= \frac{1}{\sqrt{2j+1}} \sum_{c=1}^{2j+1} \langle U|\gamma_1, j, a, c\rangle \, \langle U|\gamma_2, j, c, b\rangle$$

maximally mixed state

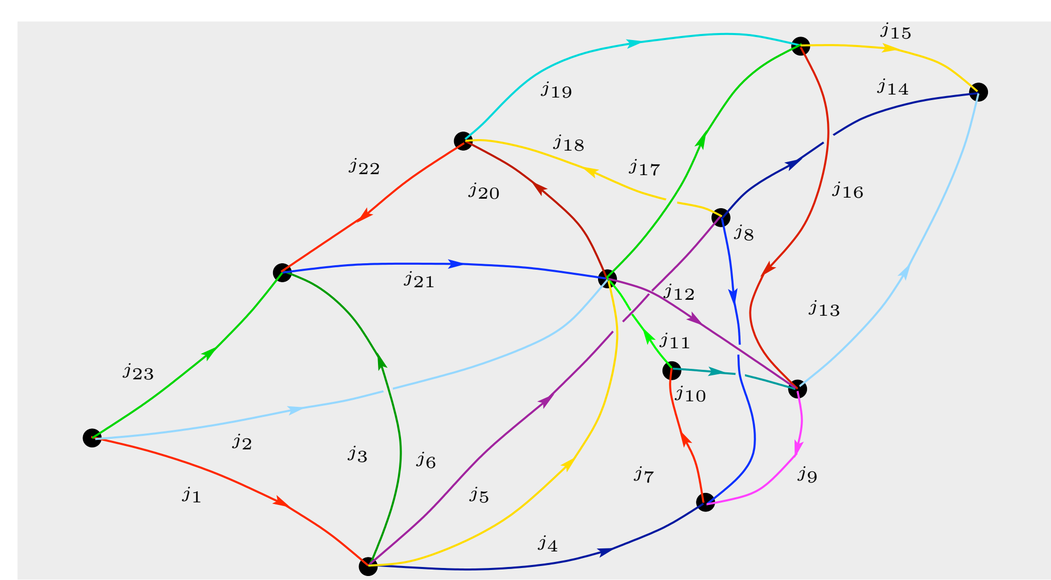




$$i_p = \frac{1}{\sqrt{2j+1}} \sum_{p=1}^{2j+1} |e_p\rangle\langle e_p|$$

$$|\mathcal{I}\rangle = \sum_{\{a,b,c\}} i^{a,b,c} |j,a\rangle \otimes |j,b\rangle \otimes |j,c\rangle$$

Donnelly, '12; Livine, Terno, '08;
Chirco, Mele, DO, Vitale, '17



Dynamics of quantum space as a group field theory

DO, '09; DO, '14

Dynamics governs gluing processes and formation of extended discrete structures

Interactions processes correspond to (simplicial) complexes in one dimension higher

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details depend on (class of) models

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"combinatorial non-locality"
in pairing of field arguments

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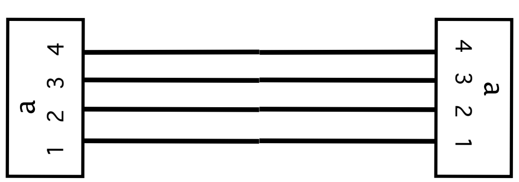
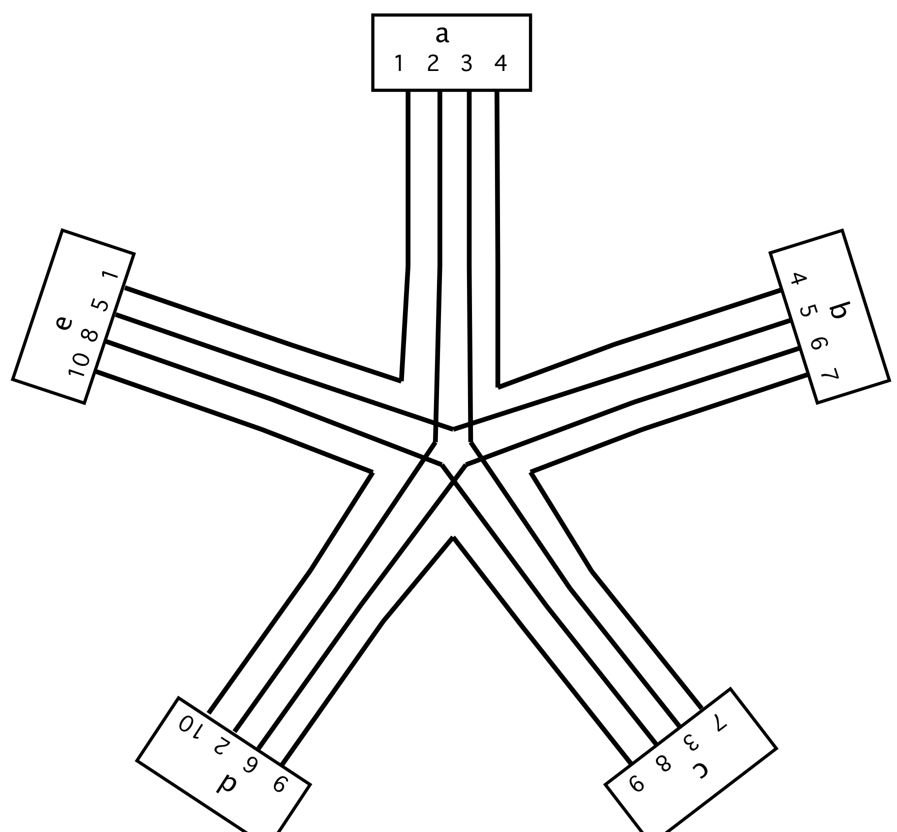
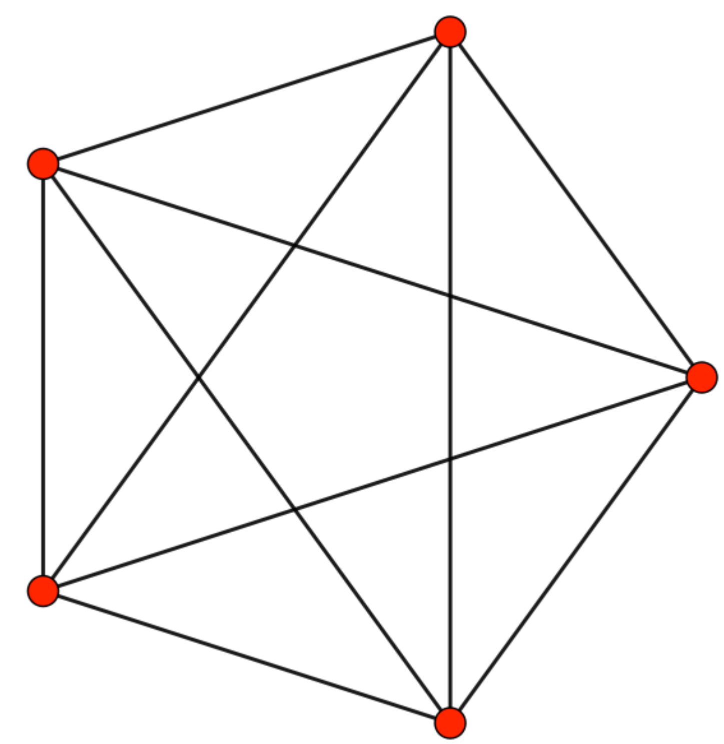
Example: simplicial interactions







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"combinatorial non-locality"
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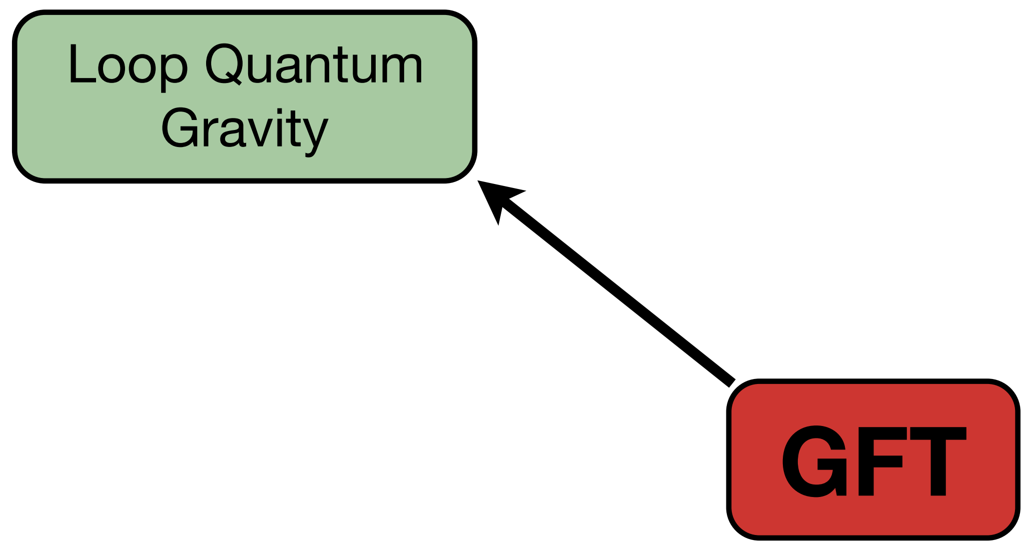
$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\mathcal{A}_\Gamma$$

Feynman diagrams = stranded diagrams dual
to cellular complexes of arbitrary topology

sum over triangulations/complexes

amplitude for each
triangulation/complex

Dynamics of quantum space as a group field theory



GFT and Loop Quantum Gravity

DO, ’13; DO, ‘14

Quantum dofs are same as in LQG (spin networks), organised in different (but similar) Hilbert space

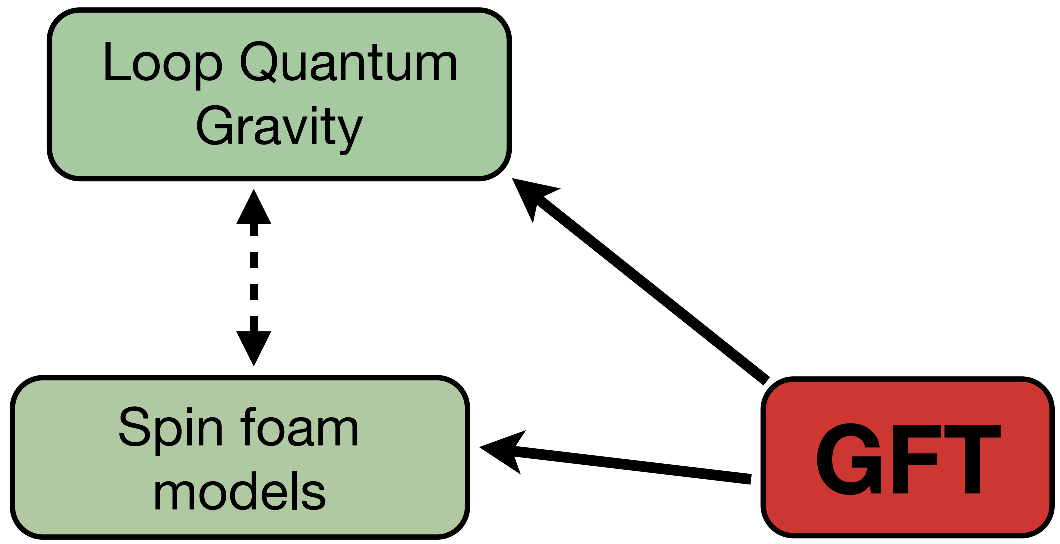
2nd quantized reformulation of states and dynamics

$$\mathcal{H}_\gamma \subset \mathcal{H}_V \qquad \Psi_\gamma(G_{ij}^{ab}) = \prod_{[(ia),(jb)]} \int_G d\alpha_{ij}^{ab}\, \phi_V(\ldots, g_{ia}\,\alpha_{ij}^{ab}, \ldots, g_{jb}\alpha_{ij}^{ab}, \ldots) = \Psi_\gamma(g_{ia}(g_{jb})^{-1})$$

$$\widehat{\mathcal{O}_{n,m}} \quad \rightarrow \quad \langle \vec{\chi}_1, ..., \vec{\chi}_m | \widehat{\mathcal{O}_{n,m}} | \vec{\chi}'_1, ..., \vec{\chi}'_n \rangle = \mathcal{O}_{n,m}\left(\vec{\chi}_1, ..., \vec{\chi}_m, \vec{\chi}'_1, ..., \vec{\chi}'_n\right) \qquad \rightarrow$$

$$\rightarrow \quad \widehat{\mathcal{O}_{n,m}}\left(\hat{\varphi}, \hat{\varphi}^\dagger\right) = \int [d\vec{g}_i][d\vec{g}'_j]\; \widehat{\varphi}^\dagger(\vec{g}_1)..\widehat{\varphi}^\dagger(\vec{g}_m)\mathcal{O}_{n,m}\left(\vec{g}_1, .., \vec{g}_m, \vec{g}'_1, .., \vec{g}'_n\right) \widehat{\varphi}(\vec{g}'_1)..\widehat{\varphi}(\vec{g}'_n)$$

Dynamics of quantum space as a group field theory



GFT and spin foam models

Spin foam model = quantum amplitude for spin network evolution

$$Z(\Gamma) = \sum_{\{J\},\{I\}|j,j',i,i'} \prod_f A_f(J,I) \prod_e A_e(J,I) \prod_v A_v(J,I)$$

$$Z(\Gamma) \leftrightarrow \begin{cases} A_f(J) \\ A_e(J,I) \\ A_v(J,I) \end{cases}$$

Reisenberger, Rovelli, '00

$$\begin{cases} \mathcal{K}(J,I) \sim \mathcal{K}(g) \\ \mathcal{V}(J,I) \sim \mathcal{V}(g) \end{cases} \quad \leftrightarrow \quad S(\varphi,\bar{\varphi})$$

Any spin foam amplitude is the Feynman amplitude of a GFT model

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\, e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\mathcal{A}_\Gamma \qquad Z(\Gamma) \equiv \mathcal{A}_\Gamma$$

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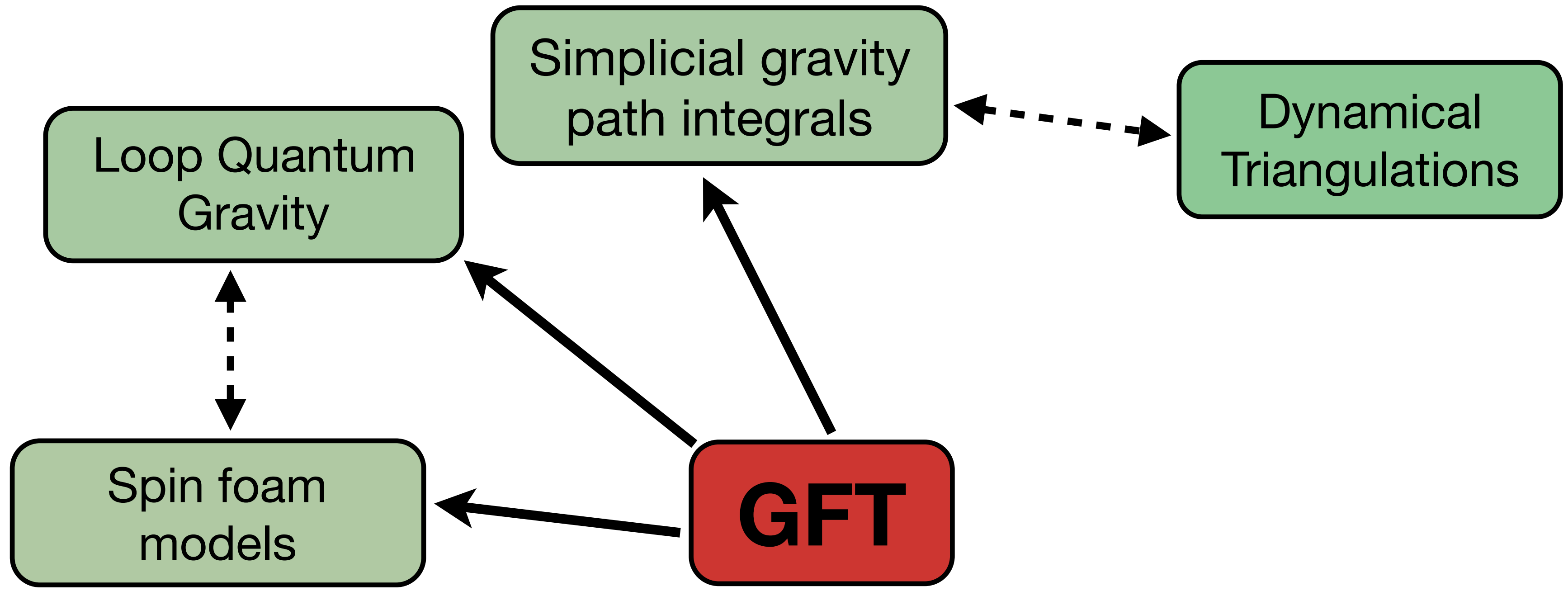

GFT and simplicial gravity path integrals

GFT Feynman amplitudes (model-dependent):
lattice gravity path integrals
(with group+Lie algebra variables)
on the lattice defined by the Feynman diagram

Baratin, DO, '11

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$

$$= \sum_{\Delta} w(\Delta)\, \mathcal{A}_\Delta = \sum_{\Delta} w(\Delta) \int \mathcal{D}g_\Delta\; e^{i\, S_\Delta(g_\Delta)} \equiv \int \mathcal{D}g\; e^{i\, S(g)}$$

dynamical triangulations + quantum Regge calculus

Dynamics of quantum space as a group field theory

quite generic:

states have interpretation in terms of quantized discrete geometries, related directly to LQG-like spin networks

(Feynman) amplitudes are discrete gravity path integrals, related directly to discrete gravity (e.g. Regge) actions

details of discrete geometric content and of quantum dynamics depend on specifics of GFT model

summary:

relation to gravity are “clear and solid” at discrete level

Continuum limit of quantum GFT dynamics

continuum limit: controlling quantum dynamics of many interacting QG dofs

GFT renormalization

Ben Geloun, Benedetti, Bonzom, Carrozza, Dittrich, Dona', Einchhorn, Gurau, Koslowski, Krajewski, Lahoche, DO, Ousmane Samary, Pereira, Riello, Rivasseau, Tanasa, Toriumi, Vitale, …

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GFT (non-perturbative) renormalization (mostly, via Functional Renormalization group techniques)

——> RG flow ~ full GFT partition function & continuum phases

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general strategy:

treat GFTs as ordinary QFTs defined on Lie group manifold

scales:
defined by propagator: e.g. spectrum of Laplacian on G = indexed by group representations

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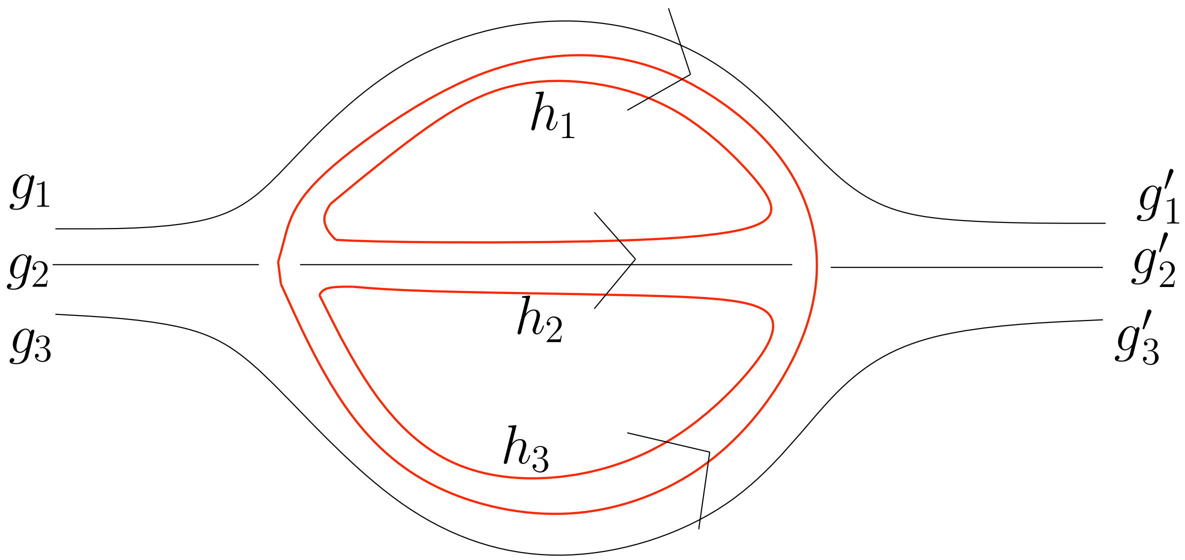
- main difficulty:
  controlling the combinatorics of GFT Feynman diagrams and interactions to control RG flow and divergences
  need to adapt/redefine many QFT notions: connectedness, subgraph contraction, Wick ordering, …..

Continuum limit of quantum GFT dynamics



example of Feynman diagram (of simplicial GFTs)

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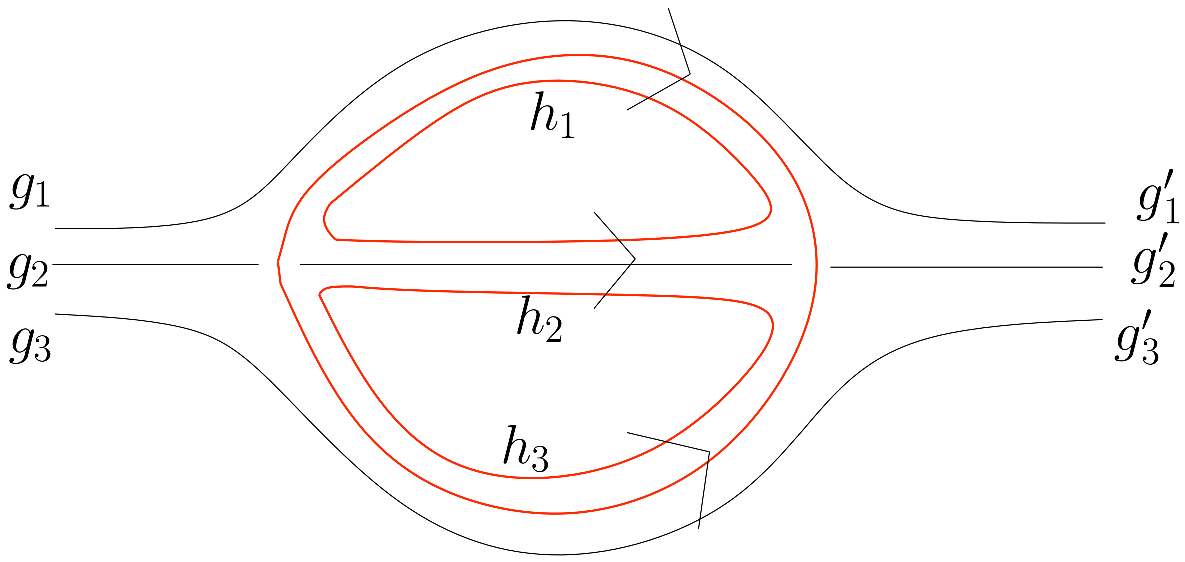

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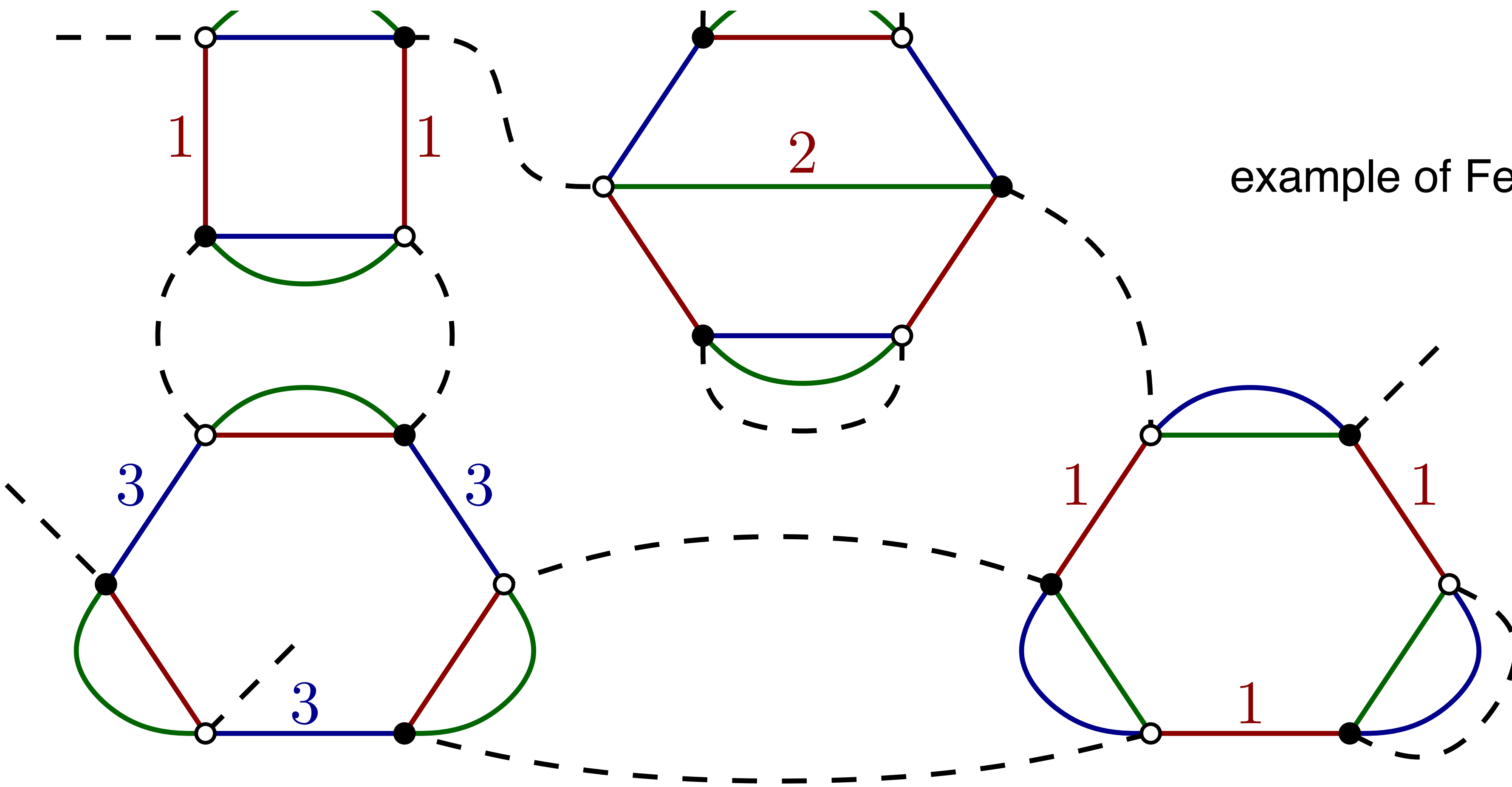

example of Feynman diagram (of Tensorial GFTs)

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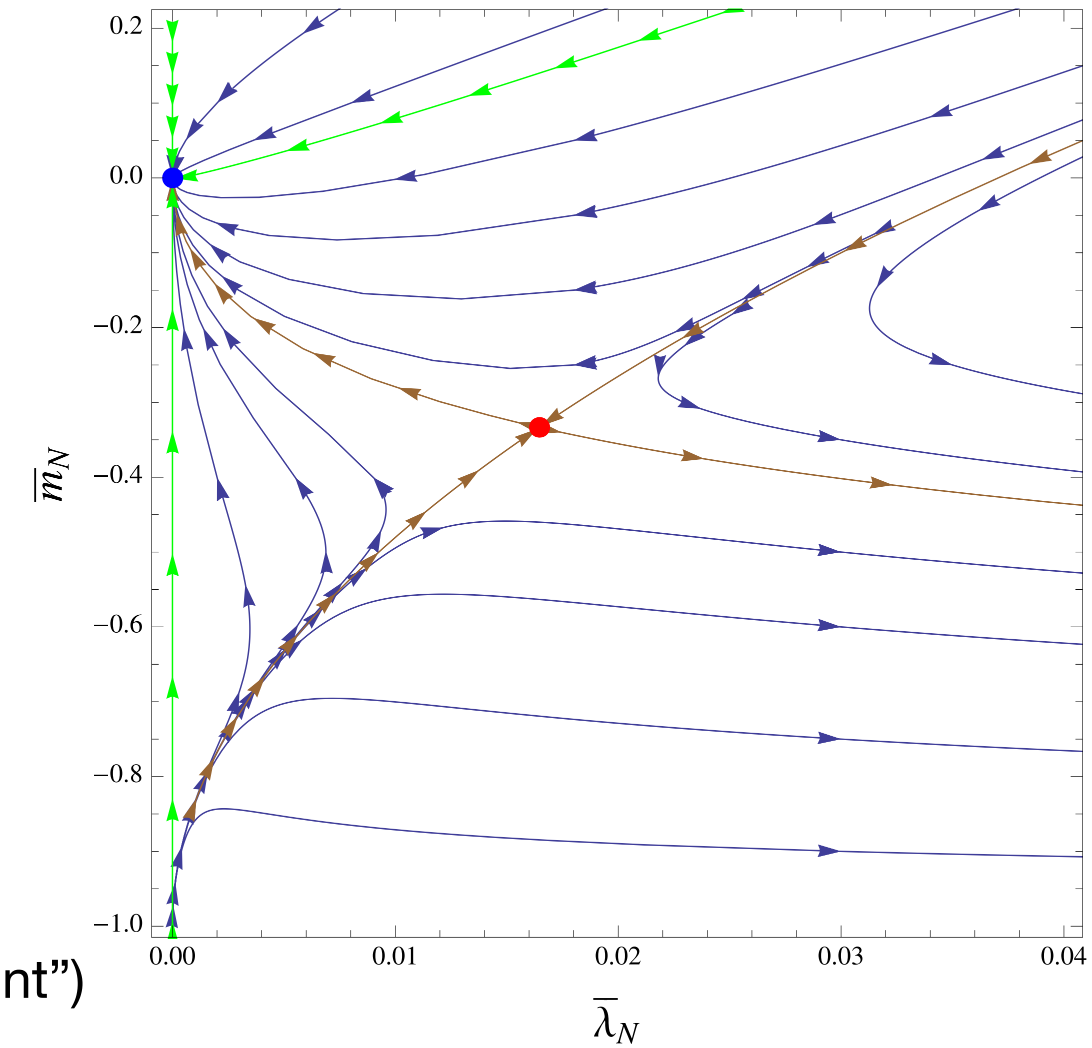

Results at perturbative level:

- Divergences in simplicial models
- Renormalizability of many TGFT models (d>2, non-abelian, w gauge invariance, ….) at all orders

Results at non-perturbative level:

- Generic asymptotic freedom/safety (“UV fixed points”)
- Non-trivial phase diagram (e.g. Wilson-Fisher “IR fixed point”)
- Hints of condensed phase (geometric, spatiotemporal phase?)

Vacuum of condensate phase:

- Field operator has non-zero expectation value ~ classical field
- classical field ~ non-trivial minimum of classical potential

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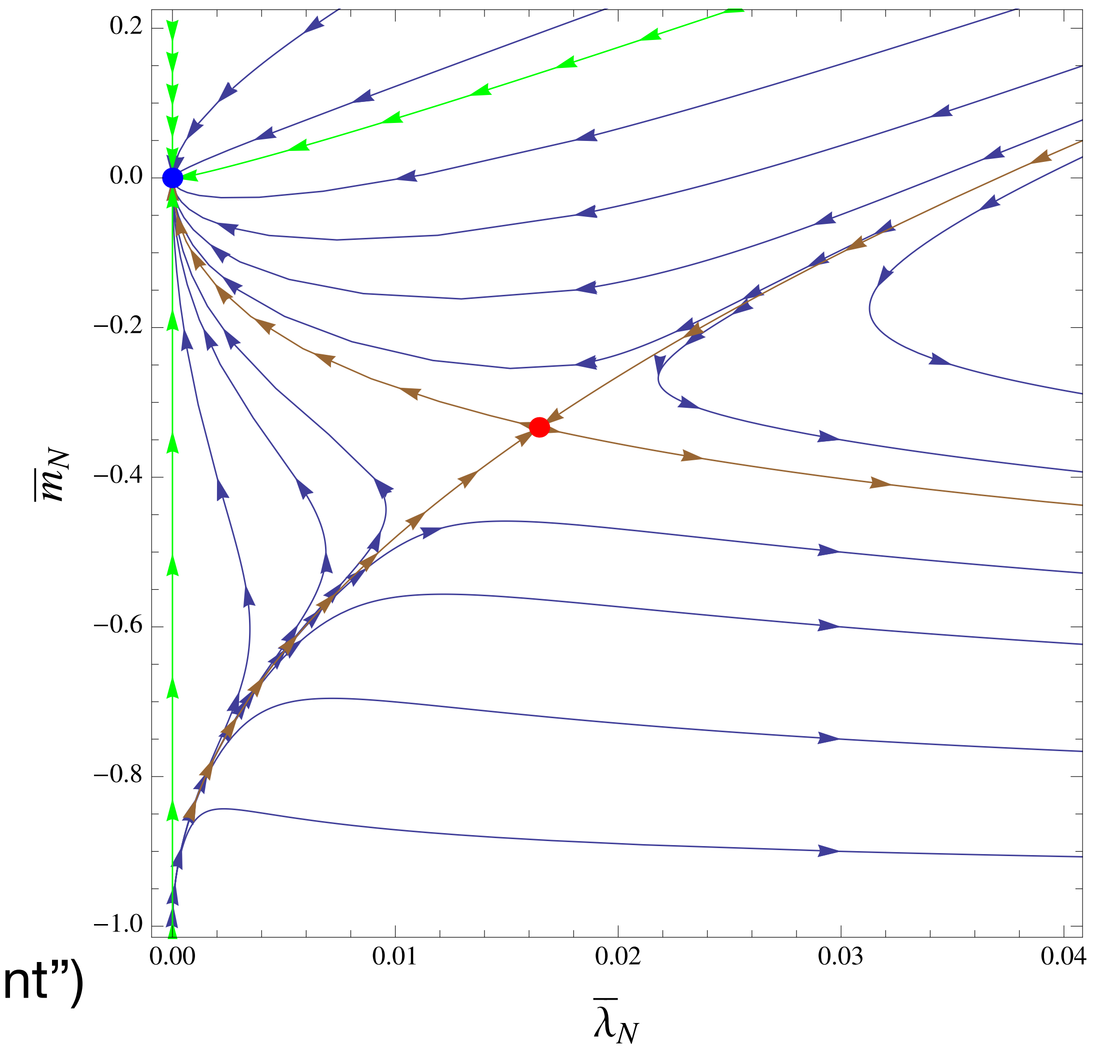

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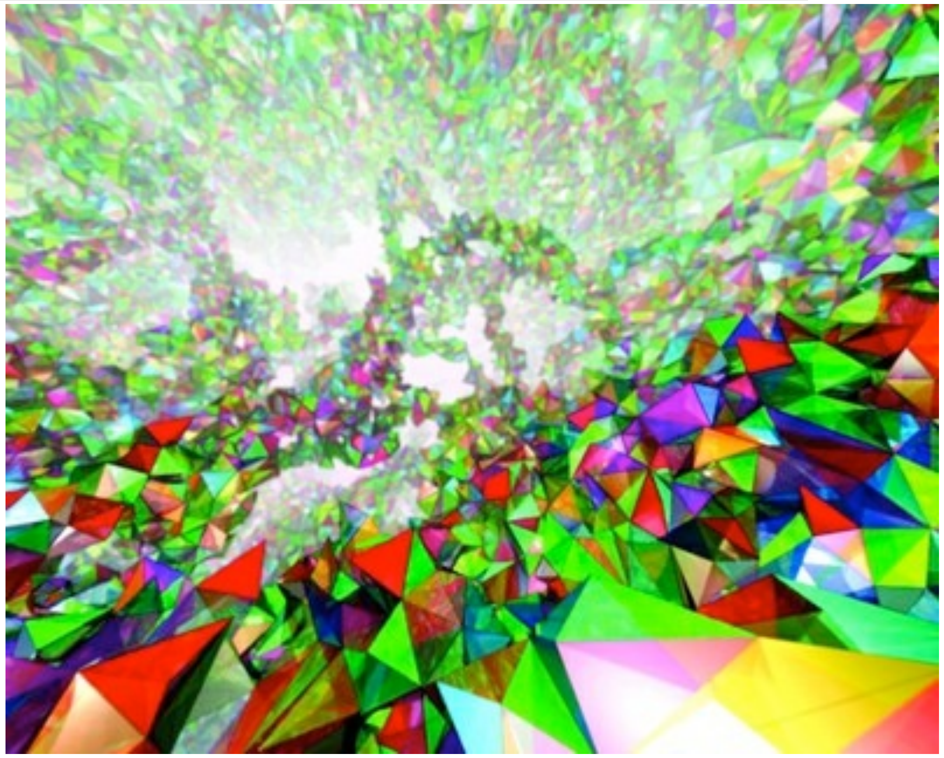
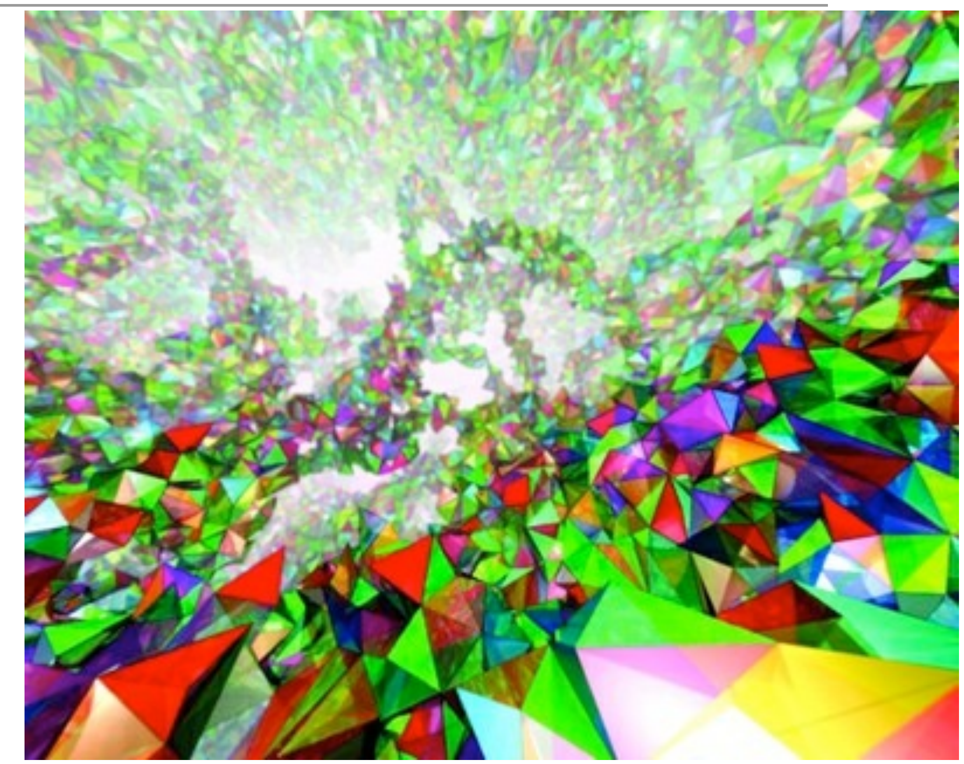
Building up continuum space and geometry

Goal: extract continuum geometric (gravitational) physics (dynamics) from QG (GFT) models



Building up continuum space and geometry



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This means:

- control QG states encoding large numbers of microscopic QG dofs
- identify those with (approximate) continuum geometric interpretation
- characterise their (geometric) properties in terms of observables
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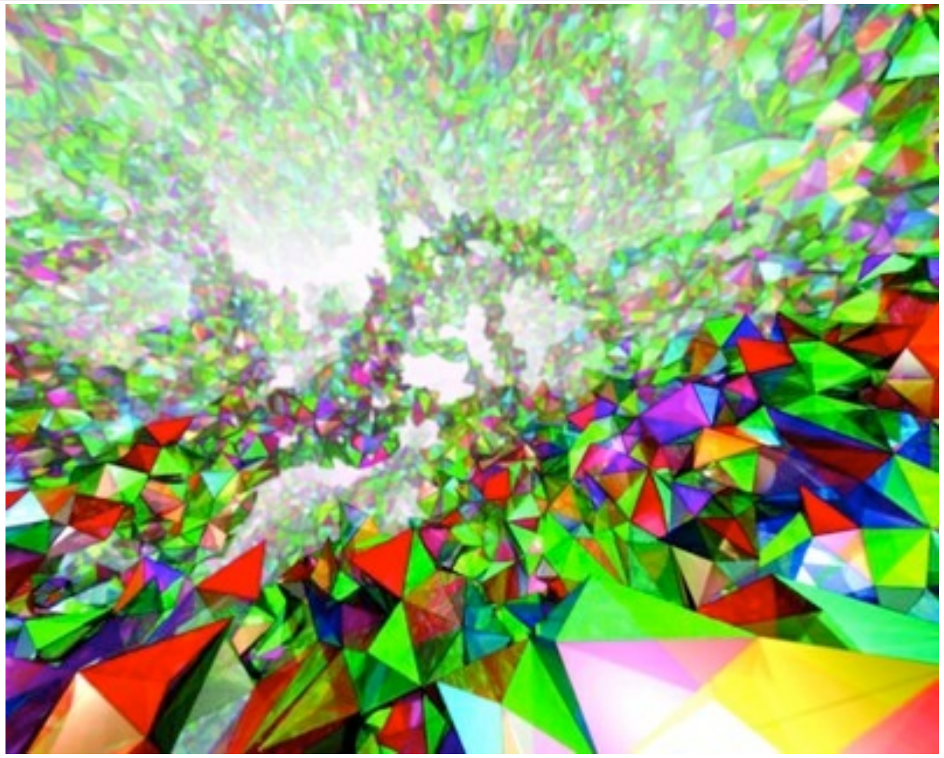

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This requires:

- controlling large graphs/complexes superpositions
- coarse graining of description
- approximations of both states, observables and dynamics

Here: take advantage of QFTformalism/methods
(universe as a quantum many-body system!)

Two points of view on cosmology

two views:

1. dynamics of (spatially) homogeneous geometries and matter fields
   (special configurations of gravitational field - homogeneous sector of General Relativity - finite # of dofs)

small number of observables, all of global nature

↓

to go beyond, quantise these geometries and fields:

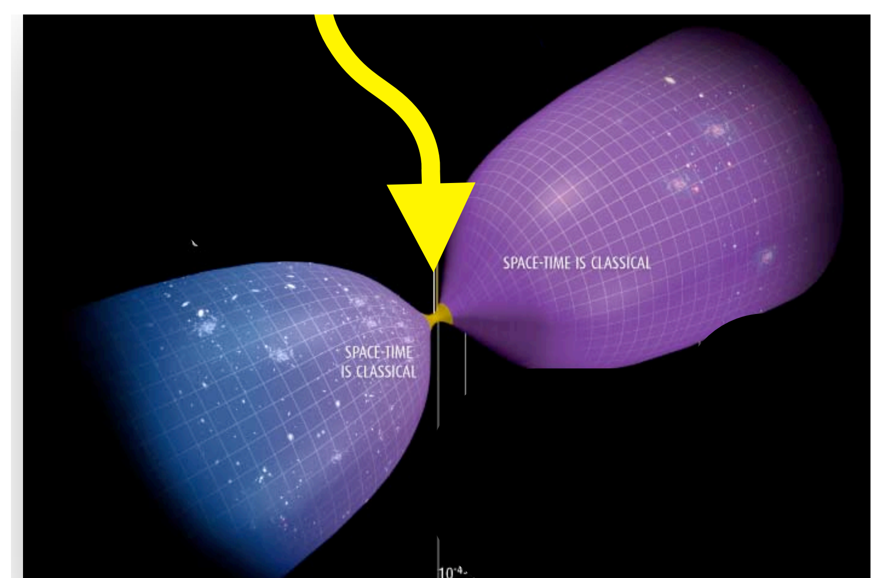
quantum cosmology (~ quantum theory of homogeneous GR)

beautiful work with lots of interesting insights

especially in Loop Quantum Cosmology (Bojowald, Ashtekar, Singh, Agullo, Pawlowski, Wilson-Ewing, …..)

e.g. big bang replaced by a bouncing scenario

just a toy model or may indeed capture features of real universe?
not derived from QG - how to embed it in full theory?



Two points of view on cosmology

two views:

1. dynamics of (spatially) homogeneous geometries
   (special configurations of gravitational field - homogeneous sector of General Relativity)

2. result of coarse graining gravitational dofs (inhomogeneities, local info) up to global quantities only

in other words: effective dynamics of
special (global) observables of full theory

this is necessarily the case if fundamental QG theory is based on non-spatiotemporal structures, and spacetime and geometry themselves are emergent

Homogeneous cosmology from full QG

heuristic

- few “macroscopic” observables, of “global” nature (understood as suitably defined averages over fundamental degrees of freedom, e.g. inhomogeneities, microscopic dofs, …)

- close to equilibrium

- insensitive to (or not too much affected by) microstructure

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hydrodynamics regime!

Cosmology as Quantum Gravity hydrodynamics

what could be the relevant hydrodynamic observables in QG? heuristic

e.g. simple averages of "one-body" observables, extensive in the "number of atoms of space"

e.g. the total volume V of space, if each "atom of space" gives a contribution to it

n.b. total volume is basic observable in homogeneous cosmology

what would key hydrodynamic quantities look like in QG?

reduced "one-body" density, function on the space of data associated with a single "atom of space"

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QG hydrodynamics ~ non-linear quantum cosmology

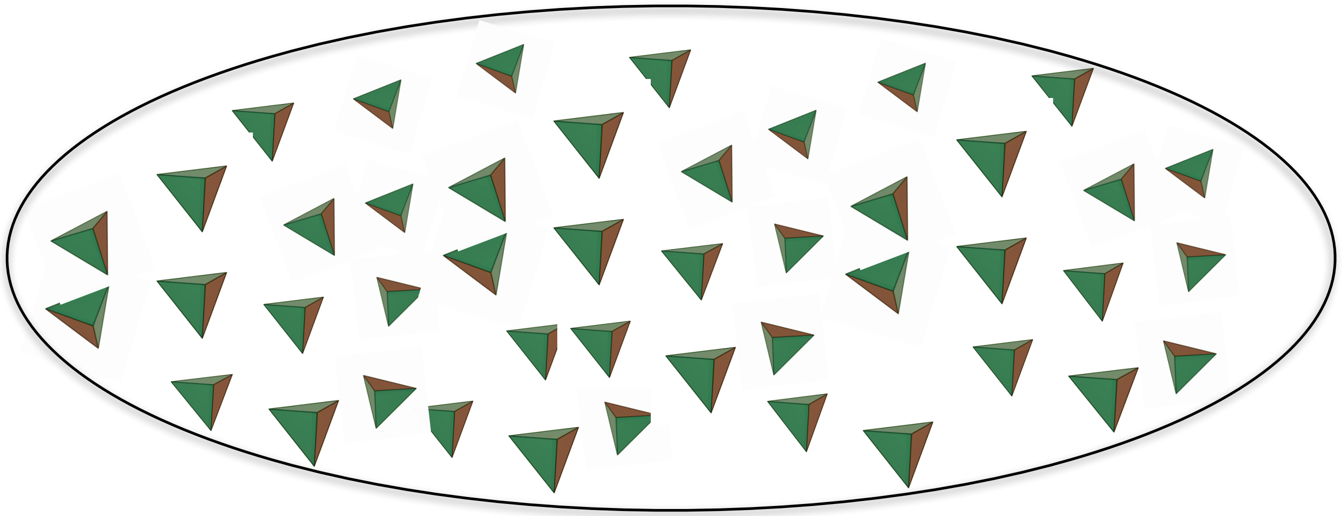
GFT condensate cosmology

Gielen, DO, Sindoni, ’13; Calcagni, De Cesare, Gielen, DO, Pithis, Sakellariadou, Sindoni, Wilson-Ewing, …

Simple GFT condensates as homogeneous continuum geometries (not encoding any topological information)

$$|\sigma\rangle := \exp\left(\hat{\sigma}\right)|0\rangle$$

$$\hat{\sigma} := \int d^4g\, \sigma(g_I)\hat{\varphi}^\dagger(g_I) \qquad \sigma(g_I k) = \sigma(g_I)$$



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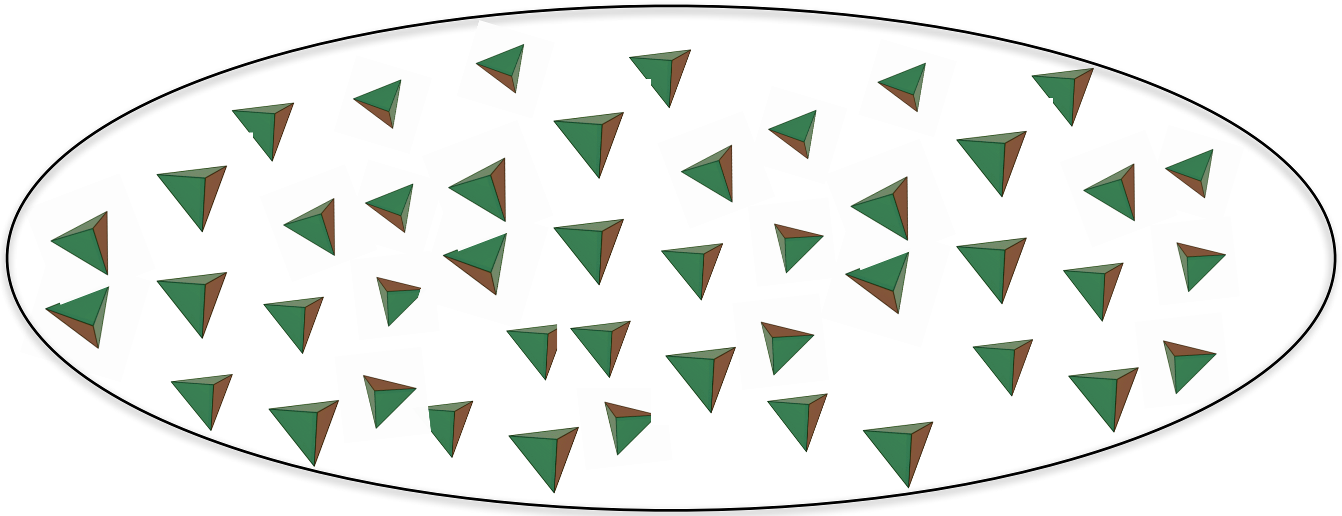

$$\sigma\left(\mathcal{D}\right)$$

described by single collective wave function
(depending on homogeneous anisotropic geometric data)

$$\begin{aligned} \mathcal{D} &\simeq \{\text{geometries of tetrahedron}\} \simeq \\ &\simeq \{\text{continuum spatial geometries at a point}\} \simeq \\ &\simeq \text{minisuperspace of homogeneous geometries} \end{aligned}$$

Gielen, ‘14

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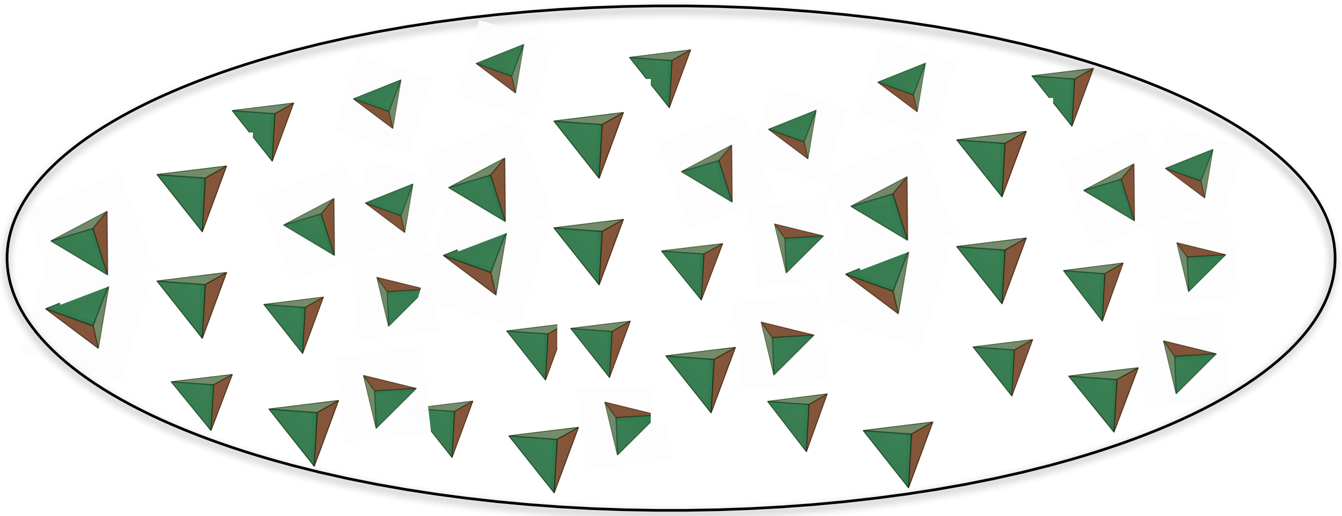

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i.e. mean field (Gross-Pitaevskii) hydrodynamics

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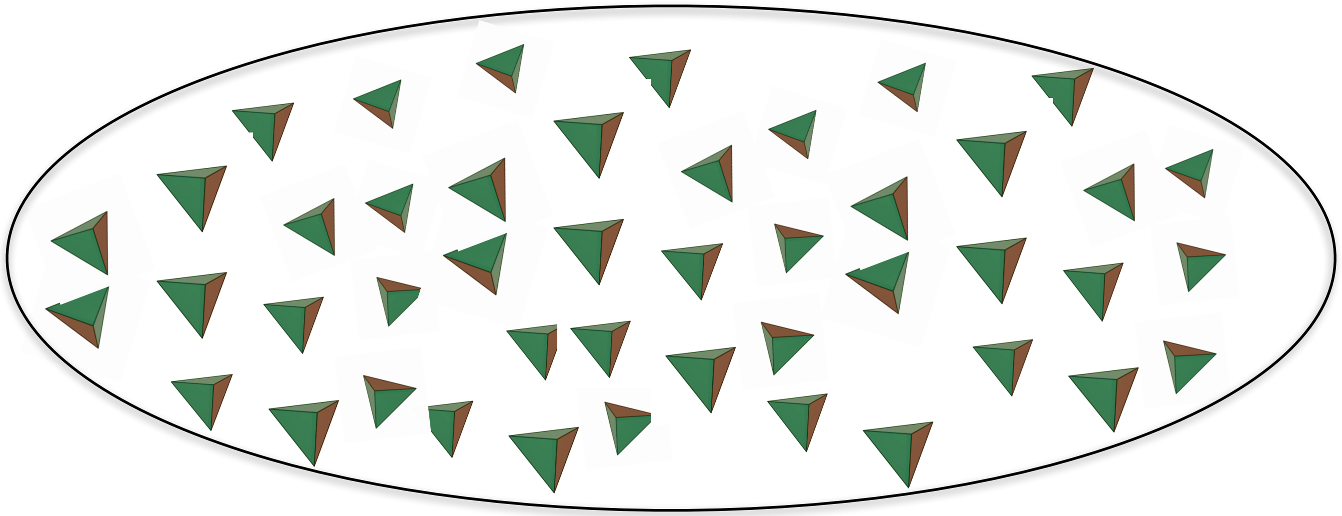

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non-linear and non-local extension of quantum cosmology-like equation for "collective wave function"

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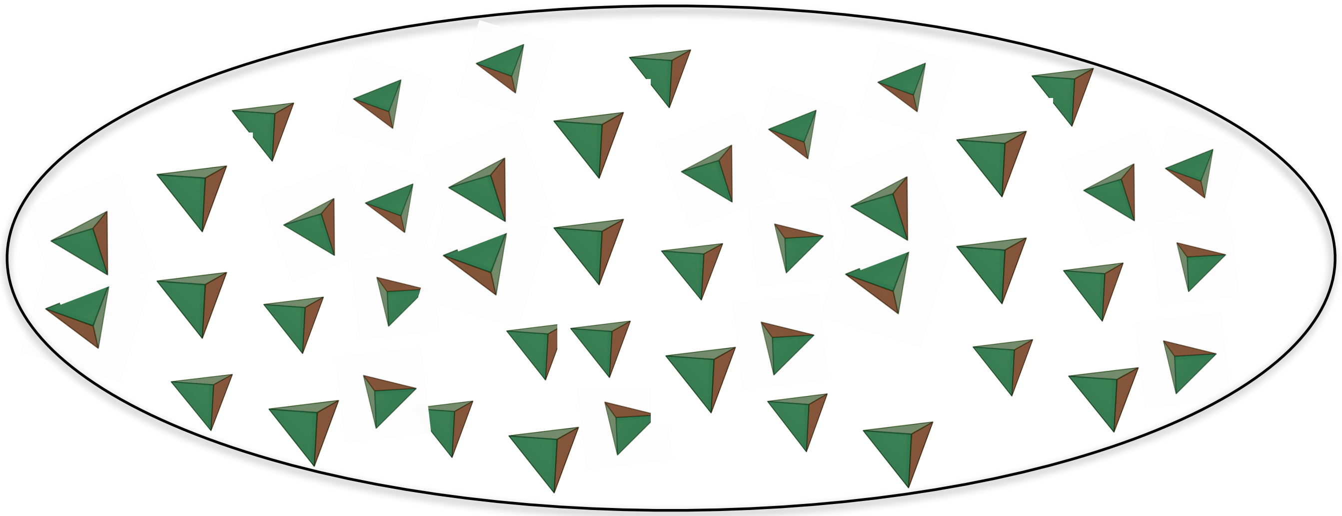

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non-linear and non-local extension of quantum cosmology-like equation for "collective wave function"

formally similar to quantum cosmology, but:

no Hilbert space structure (no superposition of "states of universe", no "collapse of cosmological wave function

"statistical nature" of wave function; still, fluctuations of all geometric quantities

GFT condensate cosmology

DO, Sindoni, Wilson-Ewing, ‘16

- (generalised) EPRL model for 4d Lorentzian QG with SU(2) data, coupled to (discretised) (pre-)scalar field

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- effective condensate hydrodynamics (non-linear quantum cosmology):

$$A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) + w_j \sigma_j(\phi)^4 = 0$$

functions A, B, w define the details of the EPRL model

GFT interaction terms sub-dominant

GFT condensate cosmology

DO, Sindoni, Wilson-Ewing, '16

GFT condensate cosmology

DO, Sindoni, Wilson-Ewing, ‘16

- key relational observables (expectation values in condensate state) with scalar field as clock:

universe volume (at fixed “time”) $$V(\phi) = \sum_j V_j \bar{\sigma}_j(\phi)\sigma_j(\phi) = \sum_j V_j \rho_j(\phi)^2 \qquad V_j \sim j^{3/2}\ell_{\rm Pl}^3$$

momentum of scalar field (at fixed “time”) $$\pi_\phi = \langle\sigma|\hat{\pi}_\phi(\phi)|\sigma\rangle = \hbar \sum_j Q_j$$

energy density of scalar field (at fixed “time”) $$\rho = \frac{\pi_\phi^2}{2V^2} = \frac{\hbar^2(\sum_j Q_j)^2}{2(\sum_j V_j \rho_j^2)^2}$$

GFT condensate cosmology

DO, Sindoni, Wilson-Ewing, '16

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observables defined in fundamental Hilbert space; intuition comes from discrete geometric interpretation of fundamental dofs; full continuum geometric interpretation emerges at collective, hydrodynamic level

Emergent bouncing cosmology from full QG

DO, Sindoni, Wilson-Ewing, '16

effective dynamics for volume - generalised Friedmann equations: (GFT interaction terms sub-dominant)

$$\boxed{\left(\frac{V'}{3V}\right)^2 = \left(\frac{2\sum_j V_j \rho_j \sqrt{E_j - \frac{Q_j^2}{\rho_j^2} + m_j^2 \rho_j^2}}{3\sum_j V_j \rho_j^2}\right)^2}$$

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$$\boxed{\frac{V''}{V} = \frac{4\sum_j V_j m_j^2 \rho_j^2}{\sum_j V_j \rho_j^2}}$$

approx. classical Friedmann eqns if $m_j^2 \approx 3G_N$

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$\Rightarrow$ $\exists j \;/\; \rho_j(\phi) \neq 0 \;\forall \phi$ $\Rightarrow$ $V = \sum_j V_j \rho_j^2$ remains positive at all times (with single turning point)

generic quantum bounce (solving classical singularity)!

M. De Cesare, M. Sakellariadou, ‘16

Special case: single spin condensate

simple condensate:

$$\sigma_j(\phi) = 0, \text{ for all } j \neq j_o$$

$$\left(\frac{V'}{3V}\right)^2 = \frac{4\pi G}{3}\left(1 - \frac{\rho}{\rho_c}\right) + \frac{V_{j_o}E_{j_o}}{9V}$$

$$\rho_c = 6\pi G\hbar^2/V_{j_o}^2 \sim (6\pi/j_o^3)\rho_{\rm Pl}$$

LQC-like modified dynamics!

DO, Sindoni, Wilson-Ewing, '16

cosmological dynamics entirely due to growth/reduction (in relational time) of number of "atoms of space"

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dominance of single-spin condensate realised in several contexts:

- mean field analysis of static GFT models in isotropic restriction: vacua strongly peaked on single spin

  A. Pithis, M. Sakellariadou, P. Tomov, '16

- mean field analysis of evolution (in relational time) of isotropic models: single spin dominates at late times
  - free GFT models (subdominant interactions) — S. Gielen, '16
  - interacting GFT models: single-spin enhanced as universe expands — A. Pithis, M. Sakellariadou, '16

Accelerated phase after bounce: QG inflation?

M. De Cesare, M. Sakellariadou, '16

M. De Cesare, A. Pithis, M. Sakellariadou, '16

for: $V = a^3$ we have:

$$\frac{\ddot{a}}{a} = \frac{1}{3}\left(\frac{\pi_\phi}{V}\right)^2\left[\frac{\partial^2_\phi V}{V} - \frac{5}{3}\left(\frac{\partial_\phi V}{V}\right)^2\right]$$

existence of accelerated expansion translates in relational time as:

$$\frac{V''}{V} > \frac{5}{3}\left(\frac{V'}{V}\right)^2$$

$$\underbrace{4m^2 + \frac{2E}{\rho^2}}_{\text{positive}} > \frac{20}{3}g^2$$

near the bounce

positive zero

detailed study of behaviour of solutions after bounce confirm a distinct accelerated phase

issue is: number of e-folds

$$N = \frac{2}{3}\log\left(\frac{\rho_{\text{end}}}{\rho_{\text{bounce}}}\right)$$

can we get at least N ~ 60?

does the acceleration last long enough (to solve cosmological problems)?

Accelerated phase after bounce: QG inflation?

M. De Cesare, A. Pithis, M. Sakellariadou, '16

- in effective cosmological dynamics neglecting GFT interactions:

$$0.119 \lesssim N \lesssim 0.186$$

acceleration is too short-lived to be physically useful

- including effects of GFT interactions (in phenomenological way):

$$S = \int \mathrm{d}\phi \, \left(A \, |\partial_\phi \sigma|^2 + \mathcal{V}(\sigma)\right)$$

$$\mathcal{V}(\sigma) = B|\sigma(\phi)|^2 + \frac{2}{n} w|\sigma|^n + \frac{2}{n'} w'|\sigma|^{n'}$$

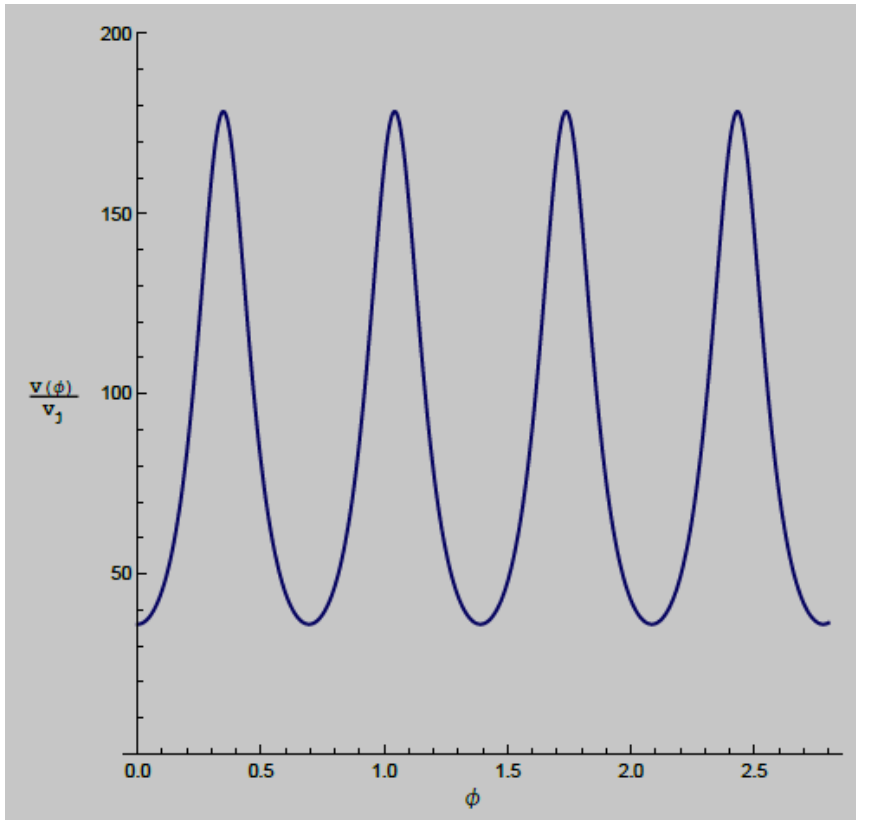
$$\sigma = \rho \, \mathrm{e}^{i\theta}$$

$$\partial_\phi^2 \rho - m^2 \rho - \frac{Q^2}{\rho^3} + \lambda \rho^{n-1} + \mu \rho^{n'-1} = 0$$

one finds:

- bounce
- accelerated expansion following bounce
- decelerated phase and recollapse

→ cyclic universe



moreover:

- N at least ~ 60
- no intermediate deceleration between beginning and end of accelerated phase

← $\lambda < 0 \text{ and } n \geq 5 \ (n' > n)$

QG-inflation from GFT condensates

Cosmological perturbations from full QG

S. Gielen, DO, '17

GFT for 4d gravity coupled to 4 free massless scalar fields used as clock and rods

+

isotropic reduction of geometric sector

$$\sigma(g_I, \phi^J) = \sum_{j=0}^{\infty} \sigma_j(\phi^J)\, \mathbf{D}^j(g_I)$$

GFT hydrodynamics equation for isotropic condensates (weak coupling)

$$\left(-B_j + A_j \partial^2_{\phi^0} + C_j\, \Delta_{\phi^i}\right) \sigma_j(\phi^J) = 0$$

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small perturbations around homogeneous condensate universes

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volume fluctuations and cosmological power spectrum (similarly for matter density fluctuations)

$$\Delta V(\phi_0, k_i; \Phi_0, K_i) \equiv \langle \hat{\tilde{V}}(\phi^0, k_i) \hat{\tilde{V}}(\Phi^0, K_i)\rangle - \langle \hat{\tilde{V}}(\phi^0, k_i)\rangle \langle \hat{\tilde{V}}(\Phi^0, K_i)\rangle$$

$$= \delta(\phi^0 - \Phi^0) \sum_j V_j^2\, |\sigma_j^0(\phi^0)|^2 \left[(2\pi)^3 \delta^3(k_i + K_i) + \epsilon\, (\tilde{\psi}_j(\phi^0, k_i + K_i) + \overline{\tilde{\psi}_j}(\phi^0, -k_i - K_i))\right]$$

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naturally approximate scale invariance

- dominant part (homogeneous condensate) exactly scale invariant
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small relative amplitude

$$\frac{\Delta V(\phi_0, k_i; \Phi_0, K_i)}{\langle \hat{V}(\phi_0)\rangle^2}$$

- dominant term ~ 1/N ~ 1/V
- perturbations further suppressed as universe expands
- if accelerated phase, further suppression of deviations from scale invariance

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different approximation (same starting point): separate universe framework $\longrightarrow$ QG-corrected eqns for long-wavelength perturbations

F. Gerhardt, DO, E. Wilson-Ewing, '18

So, what happens to the cosmological singularity?

DO, L. Sindoni, E. Wilson-Ewing, ‘16

according to (current description in) GFT condensate cosmology:

M. De Cesare, A. Pithis, M. Sakellariadou, ’16

Classical cosmological singularity is replaced by “big bounce" scenario

due to a sort of “quantum gravitational/spacetime pressure”

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more precisely:

Classical cosmological singularity is replaced by “big bounce" scenario,

in mean field restriction

of hydrodynamic approximation

within condensate phase

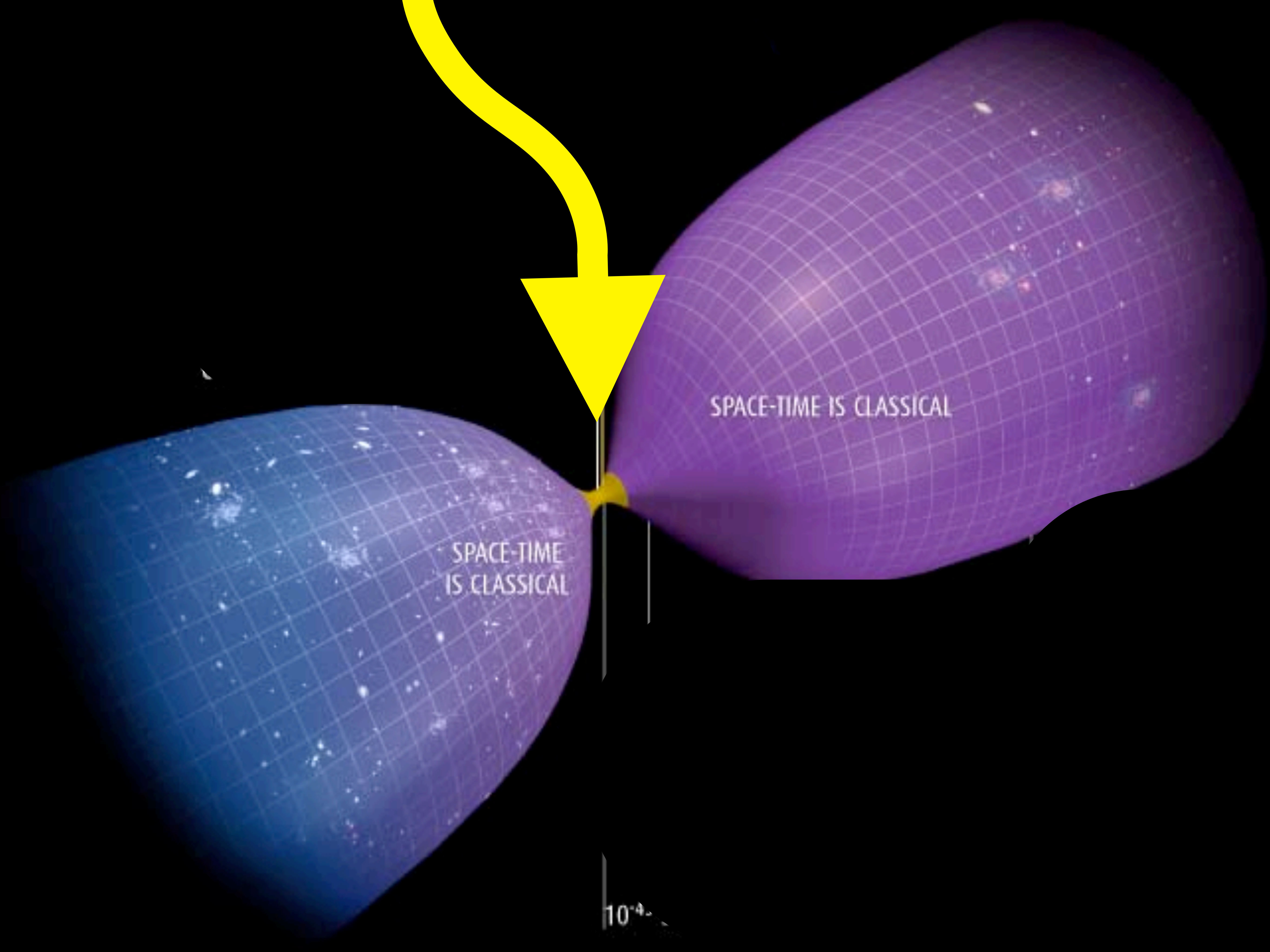
So, what happens to the cosmological singularity?

Big Bounce?

mean field approximation obviously to be improved,
leading to different condensate hydrodynamic eqns

but maybe bouncing scenario is stable under modifications of dynamics
coming from improved description of condensate hydrodynamics

then, yes, cosmic quantum bounce!


SPACE-TIME IS CLASSICAL
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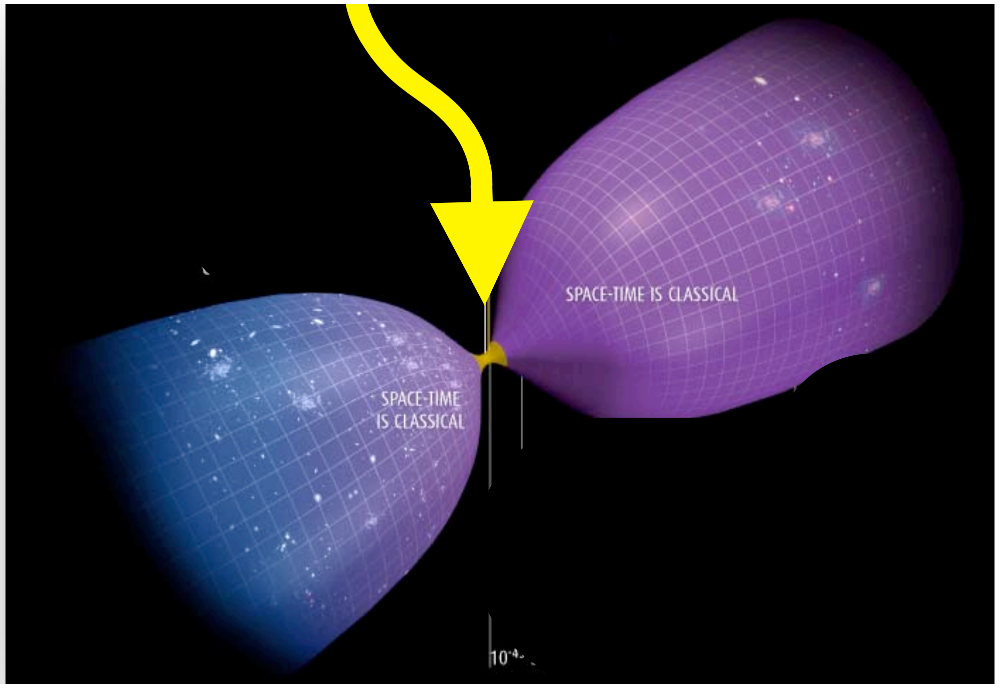
i.e. "quantum GR" regime instead of classical GR



……. if hydrodynamic approximation holds

……. if “quantum spacetime system” stays within condensate phase

So, what happens to the cosmological singularity?

If hydrodynamic approximation breaks down:

coarse-grained regime in which nice spacetime and geometric collective observables:

a) can be defined
b) give a complete characterization of the system and its dynamics (in mean value)
c) are not overwhelmed by fluctuations

can break down

because too few “atoms of space” are involved

because fluctuations become too big

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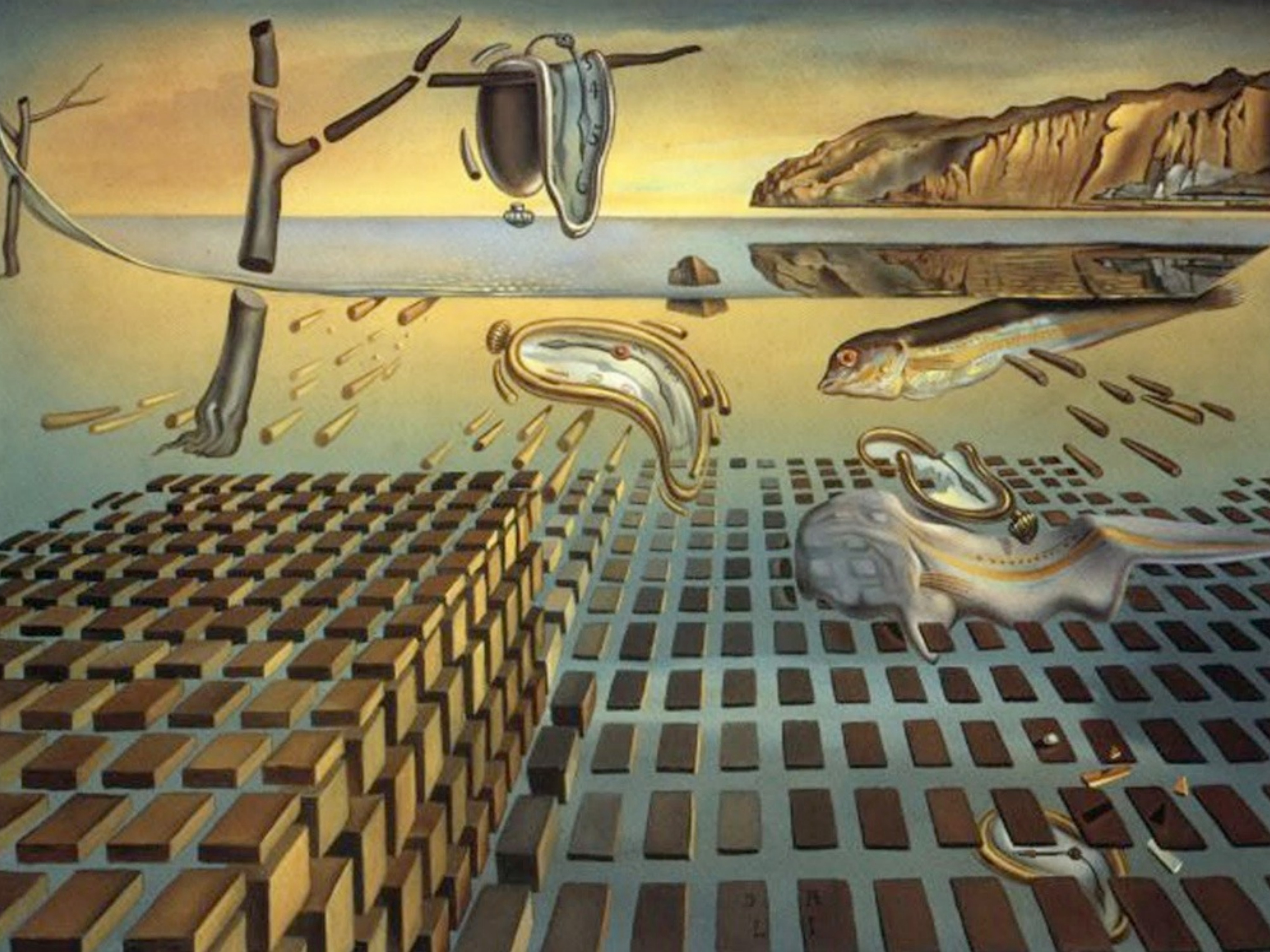

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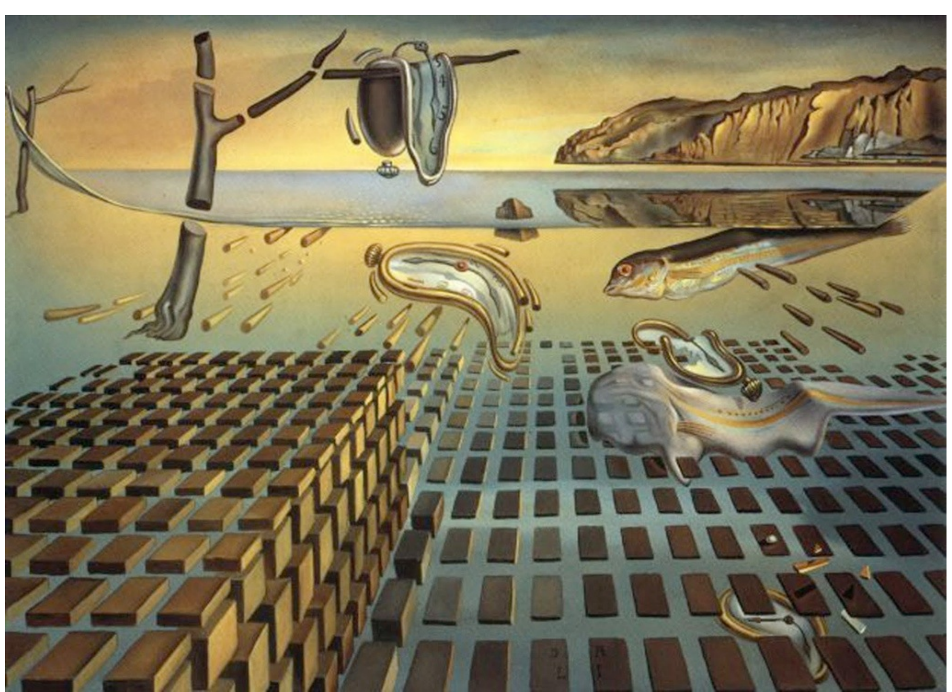

necessary: pre-geometric, non-spatiotemporal description in terms of microscopic quantum “atoms of space”

.... maybe still within the condensate phase of the “pre-geometric system”

“beyond space and time, but still within spatiotemporal phase”

So, what happens to the cosmological singularity?

If QG system leaves condensate (geometric) phase:

e.g. quantum fluctuations drive quantum dynamics (incl. couplings constants) towards phase transition, QG system reaches criticality

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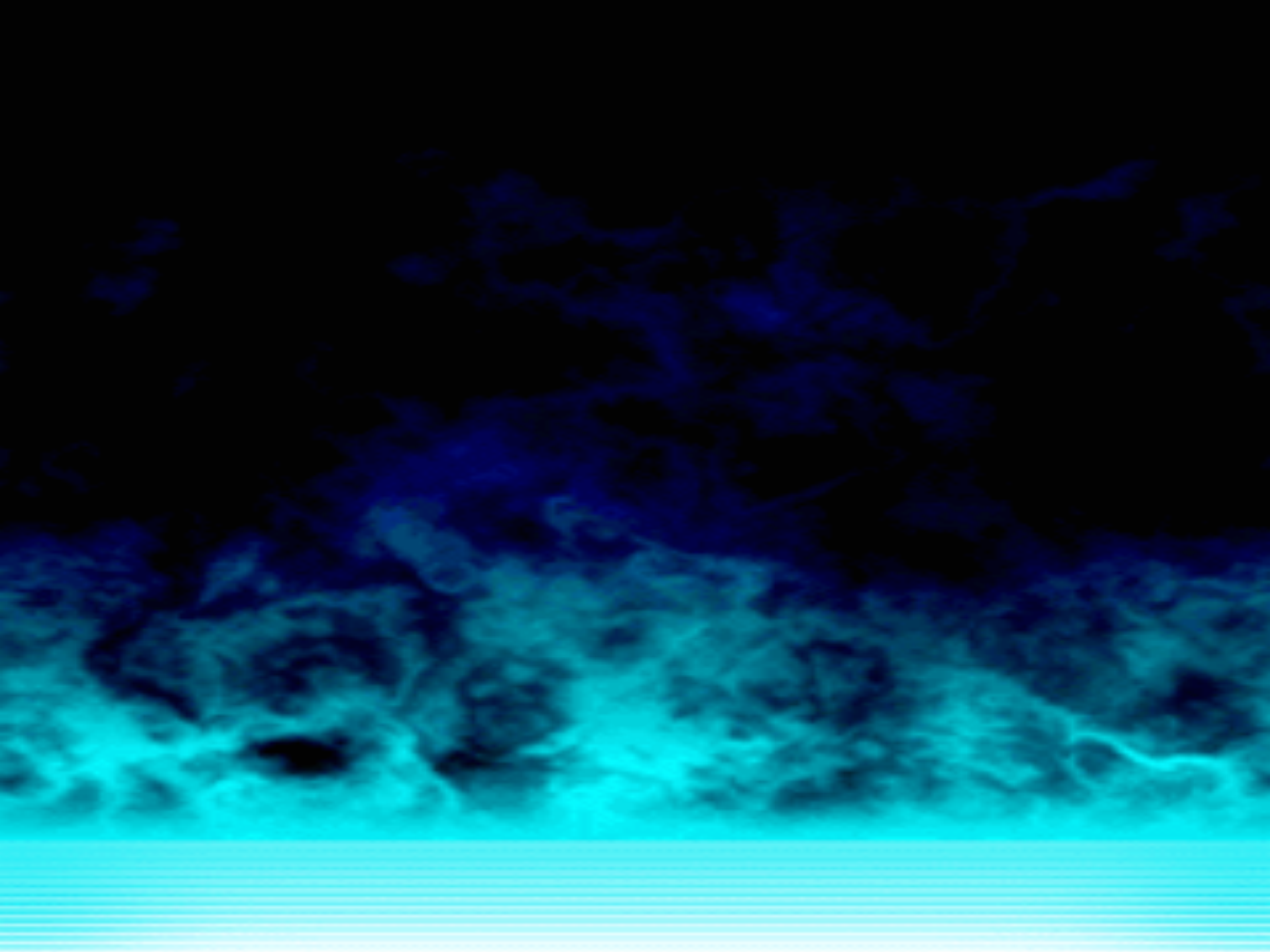
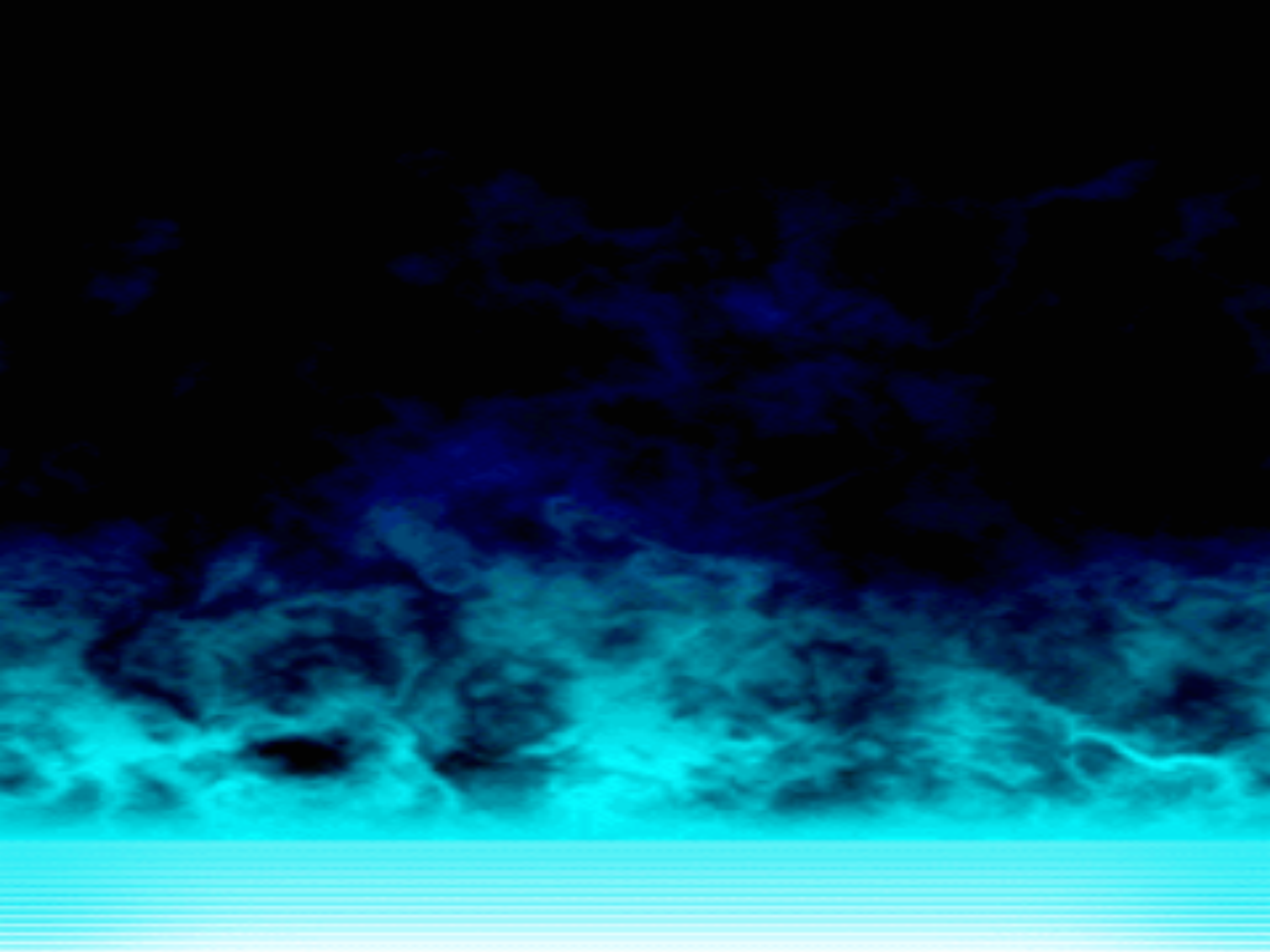
even more radical disappearance of continuum spacetime

“geometrogenesis” phase transition





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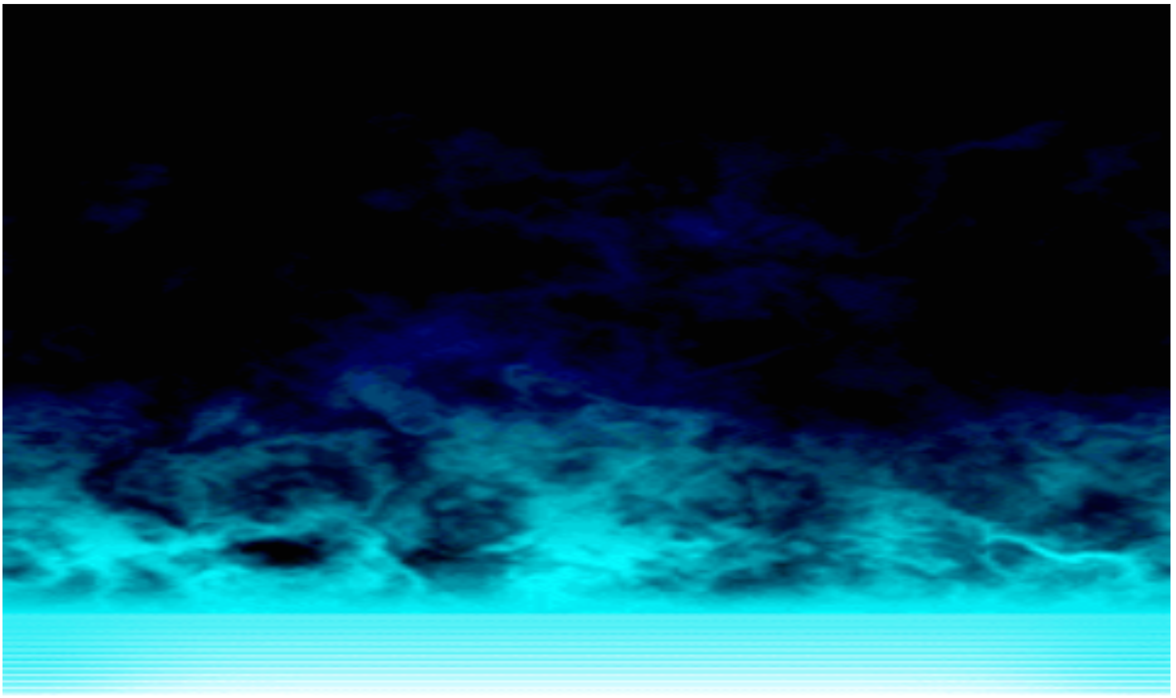

"geometrogenesis"

"beyond space and time, and out of spatiotemporal phase"

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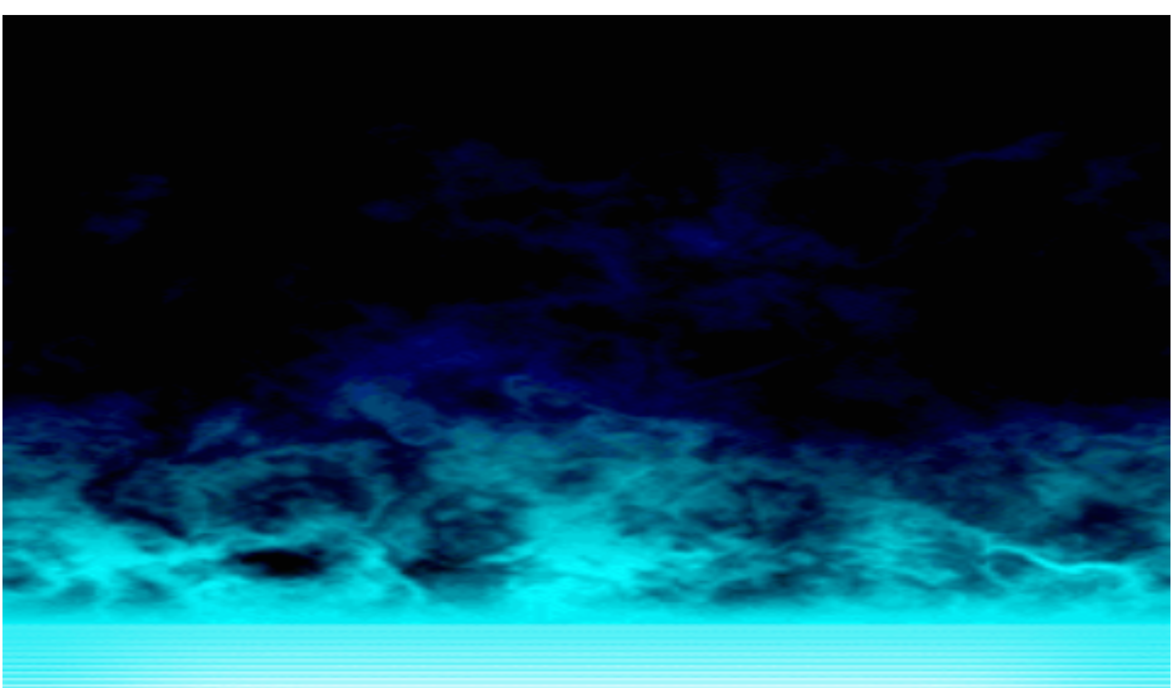
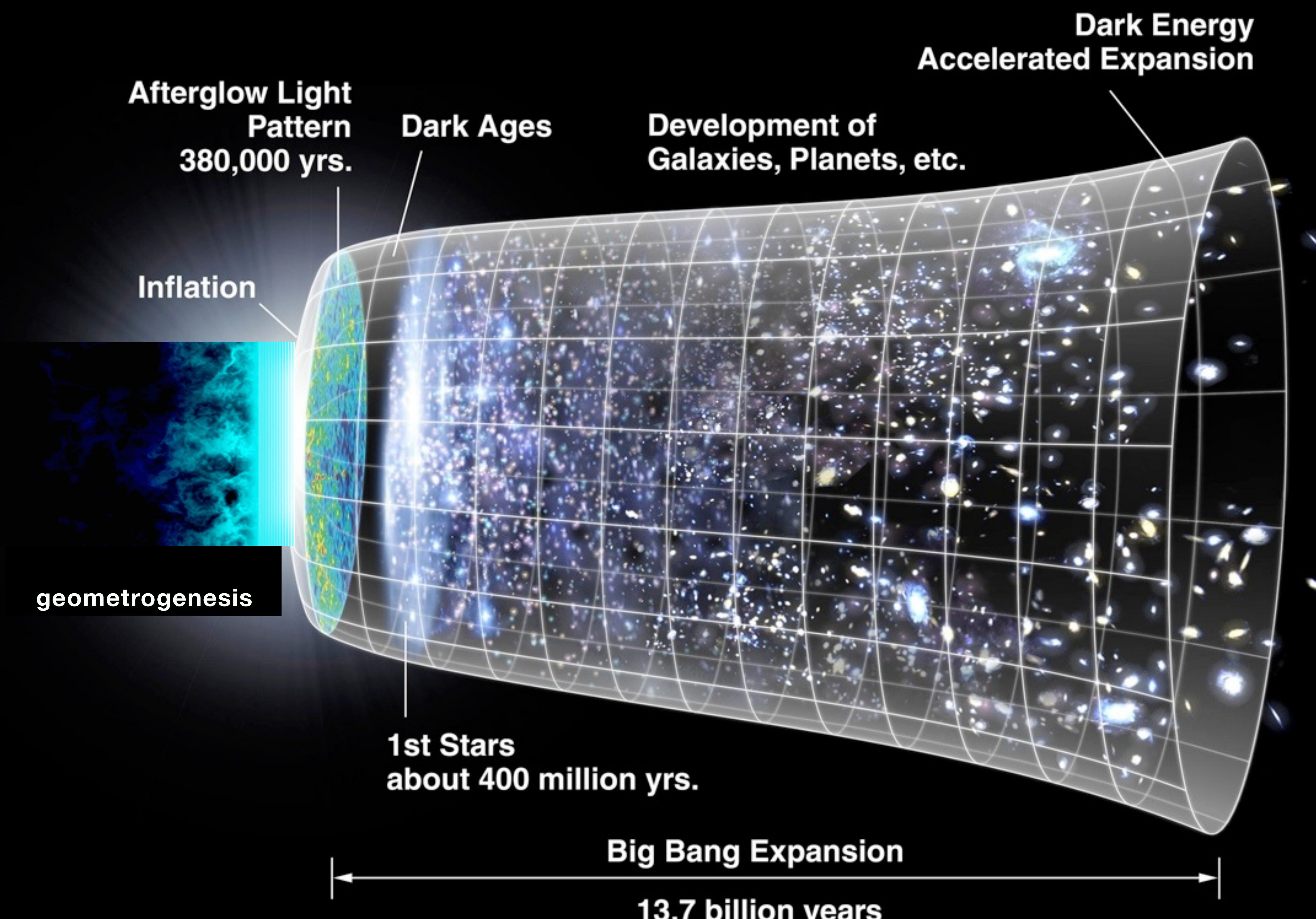
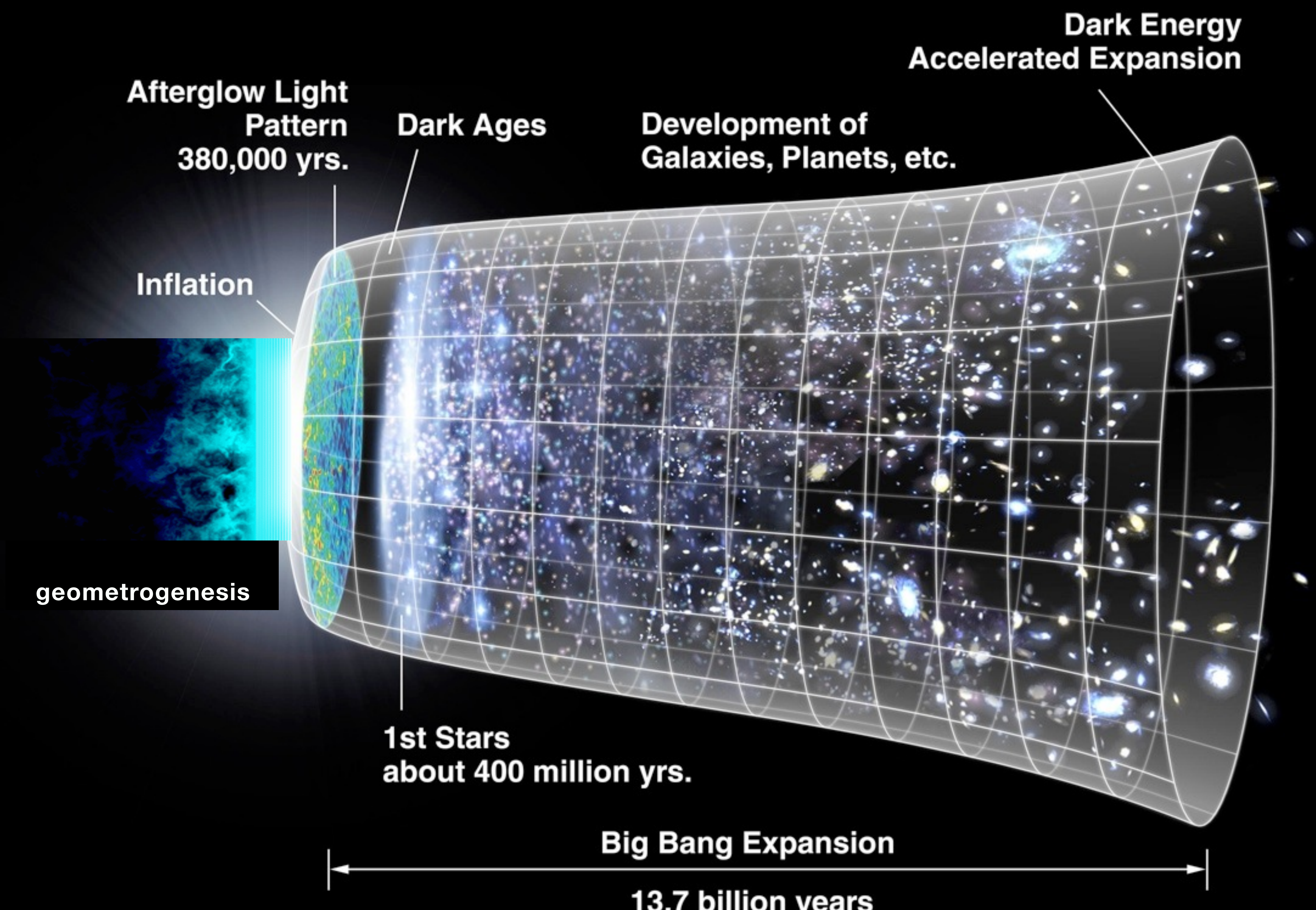


"beyond space and time, and out of spatiotemporal phase"



Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
geometrogenesis
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years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three possible resolutions of classical cosmological singularity in Quantum Gravity:

1. cosmological bounce - quantum GR replaces classical GR as dynamics of spacetime

2. disappearance of spacetime - spacetime replaced by non-spatiotemporal quantum description

3. geometrogenesis - universe crosses into non-spatiotemporal, non-geometric phase

all three scenarios (regimes of validity, dynamical steps between them, etc)

can be studied within GFT formalism for quantum gravity,

and more specifically, within GFT condensate cosmology,

using QFT techniques and methods from quantum many-body systems

Thank you for your attention