The loop quantum cosmology bounce as a Kasner transition

Edward Wilson-Ewing

University of New Brunswick

Class. Quant. Grav. **35** (2018) 065005
`arXiv:1711.10943 [gr-qc]`

Singularities in General Relativity and their Quantum Fate II
Banach Center

Loop Quantum Cosmology

In loop quantum cosmology (LQC), the non-perturbative quantization techniques of loop quantum gravity are applied to cosmological models like the Friedmann and Bianchi space-times [Bojowald; Ashtekar, Bojowald, Lewandowski; Ashtekar, Pawłowski, Singh; . . .].

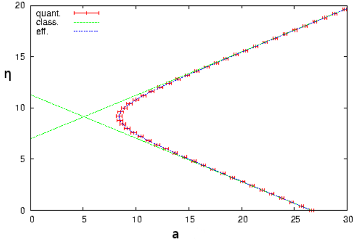
There are two key inputs in loop quantum cosmology:

- the basic operators encode areas and holonomies of the Ashtekar-Barbero connection $A_a^i = \Gamma_a^i + \gamma K_a^i$,
- the predicted Planck-scale discreteness of geometry.

In particular, for the FLRW and Bianchi I space-times the space-time curvature (which appears in the Hamiltonian constraint operator) is expressed in terms of the holonomy of A_a^i around a loop with an area of $\sim \ell_{\rm Pl}^2$.

Loop Quantum Cosmology

One of the most striking results in LQC is singularity resolution: in these space-times the singularities of general relativity are generically resolved.



[Pawłowski, Pierini, WE]

For space-times where numerical studies have been done, numerics show the singularity is replaced by a bounce, and in addition there exist 'effective equations' that provide an excellent approximation to the full quantum dynamics.

Very similar quantum gravity corrections to cosmological dynamics arise from the hydrodynamics of group field theory condensate states [Gielen, Oriti, Sindoni, WE, . . .].

The BKL Conjecture

According to the Belinski-Khalatnikov-Lifshitz (BKL) conjecture, in the approach to a space-like singularity, neighbouring points decouple and spatial derivatives become negligible in comparison to time-like derivatives.

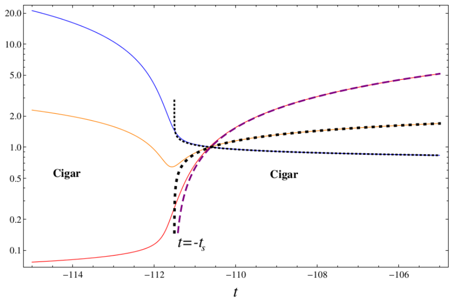
Then, the dynamics at each point is given by those of a Bianchi space-time, typically Bianchi IX.

Therefore, if the BKL conjecture is correct, the Bianchi space-times can be expected to play a central role in quantum gravity and in determining the fate of (at least some of) the singularities predicted by general relativity.

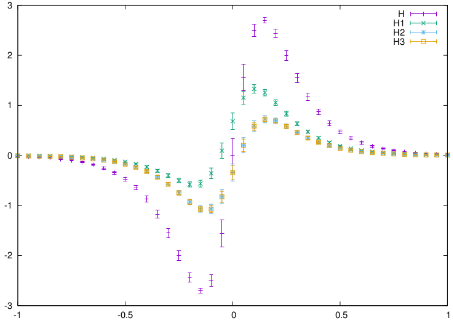
As a first step towards studying quantum gravity effects near generic singularities, I want to understand the LQC dynamics of Bianchi space-times.

LQC Dynamics for Bianchi I

Numerical solutions to the LQC effective dynamics for the Bianchi I space-time show that the singularity is replaced by a bounce [Gupt, Singh]. Studies of the full quantum dynamics confirm this [Pawłowski].



[Gupt, Singh]



[Pawłowski]

Away from the bounce, the dynamics are very well approximated by the Kasner solution from general relativity.

The LQC Bounce as a Kasner Transition

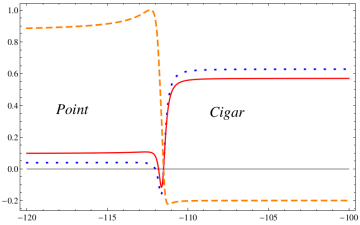
Here I will focus on the effective equations since they are much easier to study than the full quantum dynamics.

Because the degrees of freedom in LQC are heavy degrees of freedom, quantum fluctuations in semi-classical states do not increase sufficiently to become important [Rovelli, WE]. For sharply-peaked states the effective equations are expected provide a good approximation to the full quantum dynamics.

The LQC Bounce as a Kasner Transition

Here I will focus on the effective equations since they are much easier to study than the full quantum dynamics.

Because the degrees of freedom in LQC are heavy degrees of freedom, quantum fluctuations in semi-classical states do not increase sufficiently to become important [Rovelli, WE]. For sharply-peaked states the effective equations are expected provide a good approximation to the full quantum dynamics.



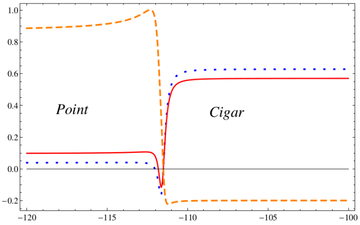
[Gupt, Singh]

Numerical solutions to the LQC effective equations show a rapid bounce, with different classical (Kasner) solutions either side [Gupt, Singh].

The LQC Bounce as a Kasner Transition

Here I will focus on the effective equations since they are much easier to study than the full quantum dynamics.

Because the degrees of freedom in LQC are heavy degrees of freedom, quantum fluctuations in semi-classical states do not increase sufficiently to become important [Rovelli, WE]. For sharply-peaked states the effective equations are expected provide a good approximation to the full quantum dynamics.



[Gupt, Singh]

Numerical solutions to the LQC effective equations show a rapid bounce, with different classical (Kasner) solutions either side [Gupt, Singh].

Could the LQC bounce be viewed as a transition between Kasner solutions?

Outline

1. Review of the Kasner Solution
2. The LQC Bounce as a Kasner Transition
3. Mixmaster Dynamics in LQC
4. Discussion

Bianchi I Variables

The line element is

$$ds^2 = -N^2 dt^2 + \sum_i a_i(t)^2 dx_i^2.$$

The basic variables I will use here are logarithmic scale factors $\alpha_i = \ln a_i$ and their conjugate momenta Π_i,

$$\{\alpha_i, \Pi_j\} = -8\pi G \delta_{ij}, \qquad \{\alpha_i, \alpha_j\} = \{\Pi_i, \Pi_j\} = 0.$$

Bianchi I Variables

The line element is

$$ds^2 = -N^2 dt^2 + \sum_i a_i(t)^2 dx_i^2.$$

The basic variables I will use here are logarithmic scale factors $\alpha_i = \ln a_i$ and their conjugate momenta Π_i,

$$\{\alpha_i, \Pi_j\} = -8\pi G \delta_{ij}, \qquad \{\alpha_i, \alpha_j\} = \{\Pi_i, \Pi_j\} = 0.$$

The Π_i are related to the scale factors by, e.g.,

$$\Pi_1 \sim a_1(\dot{a}_2 a_3 + a_2 \dot{a}_3).$$

The Hamiltonian Constraint

For the lapse $N = a_1 a_2 a_3 = \exp[\sum_i \alpha_i]$, the Hamiltonian constraint for the vacuum Bianchi I space-time is

$$\mathcal{C}_H \sim -\Pi_1\Pi_2 - \Pi_1\Pi_3 - \Pi_2\Pi_3 + \frac{1}{2}(\Pi_1^2 + \Pi_2^2 + \Pi_3^2).$$

Clearly, the Π_i are constants of the motion and

$$\alpha_1 \sim (\Pi_j + \Pi_k - \Pi_i)\tau + \alpha_1^{(0)}, \quad \text{etc.}$$

This is the full solution for the vacuum Bianchi I space-time in general relativity.

The Kasner Solution

The Kasner line element for the Bianchi I solution is

$$ds^2 = -dt^2 + (t - t_o)^{2k_1}\, dx_1^2 + (t - t_o)^{2k_2}\, dx_2^2 + (t - t_o)^{2k_3}\, dx_3^2,$$

where the k_i are the Kasner exponents.

The Kasner Solution

The Kasner line element for the Bianchi I solution is

$$ds^2 = -dt^2 + (t - t_o)^{2k_1}\, dx_1^2 + (t - t_o)^{2k_2}\, dx_2^2 + (t - t_o)^{2k_3}\, dx_3^2,$$

where the k_i are the Kasner exponents.

These can be related to the earlier solution by $dt = \exp[\sum_i \alpha_i(\tau)]d\tau$:

$$\tau \sim \frac{1}{\sum_j \Pi_j} \ln\Big(\sum_i \Pi_i (t - t_o) - \sum_i \alpha_i^{(0)} \Big),$$

and then the Kasner exponents are given by, e.g.,

$$k_1 = \frac{\Pi_2 + \Pi_3 - \Pi_1}{\Pi_1 + \Pi_2 + \Pi_3}.$$

By inspection $\sum_i k_i = 1$, and from $\mathcal{C}_H = 0$ it follows that $\sum_i k_i^2 = 1$.

Bianchi II Solution

The Hamiltonian constraint for the Bianchi II space-time is the same as for the Bianchi I space-time, with an additional exponential potential in the α_1 direction:

$$\mathcal{C}_H \sim \mathcal{C}_H^{(B.I)} + e^{4\alpha_1}.$$

The dynamics can be viewed as a Bianchi I solution bouncing once off the potential and transitioning to a different Kasner solution.

Bianchi II Solution

The Hamiltonian constraint for the Bianchi II space-time is the same as for the Bianchi I space-time, with an additional exponential potential in the α_1 direction:

$$\mathcal{C}_H \sim \mathcal{C}_H^{(B.I)} + e^{4\alpha_1}.$$

The dynamics can be viewed as a Bianchi I solution bouncing once off the potential and transitioning to a different Kasner solution.

Away from the Kasner transition, the potential is negligible and so the Π_i are constant. During the transition only Π_1 changes since the potential only depends on α_1.

During the Kasner transition, $\Pi_i \to \tilde{\Pi}_i$, with $\tilde{\Pi}_2 = \Pi_2$ and $\tilde{\Pi}_3 = \Pi_3$, while

$$\tilde{\Pi}_1 = \Pi_1 + \Delta\Pi_1.$$

It turns out that $\Delta\Pi_1$ can be found quite easily.

The Transition Rule

Away from the Kasner transition, the potential is negligible. Therefore, both Kasner solutions Π_i and $\tilde{\Pi}_i$ must each satisfy $\mathcal{C}_H^{(B.I)} = 0$. This gives the condition

$$\Delta\Pi_1 \Big[\Delta\Pi_1 - 2(\Pi_2 + \Pi_3 - \Pi_1) \Big] = 0,$$

which is satisfied by $\Delta\Pi_1 = 0$ before the transition, and by

$$\Delta\Pi_1 = 2(\Pi_2 + \Pi_3 - \Pi_1)$$

after the transition.

The Transition Rule

Away from the Kasner transition, the potential is negligible. Therefore, both Kasner solutions Π_i and $\tilde{\Pi}_i$ must each satisfy $\mathcal{C}_H^{(B.I)} = 0$. This gives the condition

$$\Delta\Pi_1 \Big[\Delta\Pi_1 - 2(\Pi_2 + \Pi_3 - \Pi_1)\Big] = 0,$$

which is satisfied by $\Delta\Pi_1 = 0$ before the transition, and by

$$\Delta\Pi_1 = 2(\Pi_2 + \Pi_3 - \Pi_1)$$

after the transition.

Putting this result into the relation between the Kasner exponents and the Π_i gives the well-known Kasner transition map for the Kasner exponents [Belinski, Khalatnikov, Lifshitz]:

$$\tilde{k}_1 = \frac{-k_1}{1+2k_1}, \qquad \tilde{k}_2 = \frac{k_2 + 2k_1}{1+2k_1}, \qquad \tilde{k}_3 = \frac{k_3 + 2k_1}{1+2k_1}.$$

Bianchi I LQC Effective Dynamics

The LQC effective dynamics for the vacuum Bianchi I space-time are generated by the Hamiltonian constraint (for $N = \exp[\sum_i \alpha_i] \sim V$)

[Chiou, Vandersloot; Ashtekar, WE]

$$\mathcal{C}_H \sim -\frac{V^2}{\Delta} \sum_{i \neq j} \sin \bar{\mu}_i \gamma K_i \sin \bar{\mu}_j \gamma K_j$$

$$\sim -\frac{V^2}{\Delta} \sum_{i \neq j \neq k} \sin \frac{\gamma \sqrt{\Delta}}{2V} (\Pi_j + \Pi_k - \Pi_i) \sin \frac{\gamma \sqrt{\Delta}}{2V} (\Pi_i + \Pi_k - \Pi_j),$$

with $\bar{\mu}_1 = \sqrt{p_1 \Delta / p_2 p_3} = \sqrt{\Delta}\, p_1 / V$, while γ is the Barbero-Immirzi parameter and Δ is the minimal non-zero eigenvalue of the LQG area operator.

An important point here is that α_i enters $\mathcal{C}_H$ only in terms of the combination $V \sim \exp[\sum_i \alpha_i]$.

Dynamics for Π_i in LQC

Note that from $\{\alpha_i, \Pi_j\} = -8\pi G \delta_{ij}$, it follows that

$$V \sim \exp[\sum_i \alpha_i], \qquad \{\Pi_i, V\} = 8\pi G V,$$

and as a result the equation of motion for the Π_i are identical since $\mathcal{C}_H$ depends on α_i only through the combination V:

$$\frac{d\Pi_i}{d\tau} = \{\Pi_i, \mathcal{C}_H\} = 8\pi G V \, \frac{d\mathcal{C}_H}{dV}.$$

Dynamics for Π_i in LQC

Note that from $\{\alpha_i, \Pi_j\} = -8\pi G\delta_{ij}$, it follows that

$$V \sim \exp[\sum_i \alpha_i], \qquad \{\Pi_i, V\} = 8\pi GV,$$

and as a result the equation of motion for the Π_i are identical since $\mathcal{C}_H$ depends on α_i only through the combination V:

$$\frac{d\Pi_i}{d\tau} = \{\Pi_i, \mathcal{C}_H\} = 8\pi GV \frac{d\mathcal{C}_H}{dV}.$$

Well before the LQC bounce the Π_i are constant (since LQC effects are negligible), and after the bounce the new $\tilde{\Pi}_i$ are also constant. Since their equations of motion are identical, they are all shifted by the same amount $\Delta\Pi$ by the LQC bounce:

$$\tilde{\Pi}_i = \Pi_i + \Delta\Pi.$$

Transformation Rule for Kasner Exponents

Since the Π_i and $\tilde{\Pi}_i$ each correspond to a classical Kasner solution away from the bounce (respectively far before and far after), both must satisfy the general relativity Hamiltonian constraint $\mathcal{C}_H = 0$. This constrains $\Delta\Pi$:

$$\Delta\Pi \left(2 \sum_i \Pi_i + 3\Delta\Pi \right) = 0.$$

There are two solutions: $\Delta\Pi = 0$ corresponding to the pre-bounce solution, and the post-bounce solution

$$\Delta\Pi = -\frac{2}{3}\Big(\Pi_1 + \Pi_2 + \Pi_3\Big).$$

Transformation Rule for Kasner Exponents

Since the Π_i and $\tilde{\Pi}_i$ each correspond to a classical Kasner solution away from the bounce (respectively far before and far after), both must satisfy the general relativity Hamiltonian constraint $\mathcal{C}_H = 0$. This constrains $\Delta\Pi$:

$$\Delta\Pi \left(2 \sum_i \Pi_i + 3\Delta\Pi \right) = 0.$$

There are two solutions: $\Delta\Pi = 0$ corresponding to the pre-bounce solution, and the post-bounce solution

$$\Delta\Pi = -\frac{2}{3}\Big(\Pi_1 + \Pi_2 + \Pi_3\Big).$$

In terms of the Kasner exponents, this gives

$$k_i \to \tilde{k}_i = \frac{2}{3} - k_i,$$

which agrees with the results of numerical simulations [Gupt, Singh].

The LQC Bounce as a Kasner Transition

So the loop quantum cosmology bounce acts, in effect, as a rapid transition between two Kasner solutions with the simple transition rule

$$k_i \to \tilde{k}_i = \frac{2}{3} - k_i$$

relating the values of the Kasner exponents before and after the LQC bounce.

Bianchi IX Solution

The Hamiltonian constraint for the Bianchi IX space-time has multiple exponential potentials:

$$\mathcal{C}_H \sim \mathcal{C}_H^{(B.I)} + e^{4\alpha_1} + e^{4\alpha_2} + e^{4\alpha_3},$$

keeping only the dominant terms in the potential for large curvatures, note these are three copies of the Bianchi II potential.

The Bianchi IX space-time can be viewed as a sequence of Kasner solutions (or 'epochs'). In the vacuum case there are an infinite number of Kasner epochs during the approach to the singularity.

Bianchi IX Solution

The Hamiltonian constraint for the Bianchi IX space-time has multiple exponential potentials:

$$\mathcal{C}_H \sim \mathcal{C}_H^{(B.I)} + e^{4\alpha_1} + e^{4\alpha_2} + e^{4\alpha_3},$$

keeping only the dominant terms in the potential for large curvatures, note these are three copies of the Bianchi II potential.

The Bianchi IX space-time can be viewed as a sequence of Kasner solutions (or 'epochs'). In the vacuum case there are an infinite number of Kasner epochs during the approach to the singularity.

During each Kasner epoch, it is convenient to order the Kasner exponents as

$$-\tfrac{1}{3} < k_{\rm min} < 0 < k_{\rm mid} < \tfrac{2}{3} < k_{\rm max} < 1,$$

and then to denote their respective logarithmic scale factors by $\alpha_{\rm min}, \alpha_{\rm mid}, \alpha_{\rm max}$.

Kasner Parameters

Each Kasner epoch can be parametrized by

$$u = \frac{k_{\rm max}}{k_{\rm mid}}, \qquad p_\Omega \sim \sum_i \Pi_i,$$

$$v = \frac{k_{\rm mid}}{k_{\rm max}} \cdot \frac{k_{\rm min}\alpha_{\rm max} - k_{\rm max}\alpha_{\rm min}}{k_{\rm min}\alpha_{\rm mid} - k_{\rm mid}\alpha_{\rm min}} + 1, \qquad \kappa = k_{\rm max}\left(\frac{\alpha_{\rm min}}{k_{\rm min}} - \frac{\alpha_{\rm mid}}{k_{\rm mid}}\right),$$

which are constant for a Kasner solution.

At each transition between two Kasner epochs, following from the Bianchi II transformation rule for the Kasner exponents given earlier:

$$u \to \tilde{u} = \begin{cases} u - 1 & \text{if} \quad u - 1 > 1, \\ (u-1)^{-1} & \text{otherwise,} \end{cases}$$

$$p_\Omega \to \tilde{p}_\Omega = p_\Omega \, \frac{u^2 - u + 1}{u^2 + u + 1}.$$

Transition Rules

Simple (although approximate) transition rules for v and κ can be derived under the assumption that the transition between subsequent Kasner epochs occurs instantaneously when the largest $\alpha_i = 0$.

Then,

$$v \to \tilde{v} = \begin{cases} v+1 & \text{if} \quad u-1>1, \\ 1+1/v & \text{otherwise,} \end{cases}$$

$$\kappa \to \tilde{\kappa} = \begin{cases} \kappa & \text{if} \quad u-1>1, \\ v\,\kappa/(u-1) & \text{otherwise.} \end{cases}$$

Transition Rules

Simple (although approximate) transition rules for v and κ can be derived under the assumption that the transition between subsequent Kasner epochs occurs instantaneously when the largest $\alpha_i = 0$.

Then,

$$v \to \tilde{v} = \begin{cases} v + 1 & \text{if} \quad u - 1 > 1, \\ 1 + 1/v & \text{otherwise}, \end{cases}$$

$$\kappa \to \tilde{\kappa} = \begin{cases} \kappa & \text{if} \quad u - 1 > 1, \\ v\,\kappa/(u-1) & \text{otherwise}. \end{cases}$$

Can analogous transition rules for (u, v, p_Ω, κ) be derived for the LQC bounce?

Spatial Curvature in LQC

In the presence of spatial curvature, the holonomy of the field strength around a loop of minimal area is not almost-periodic in the connection, so a different approach is required.

So far, there are two suggestions: 'A' and 'K' loop quantizations based on the parallel transport of the Ashtekar-Barbero connection and the extrinsic curvature respectively [Vandersloot; Ashtekar, WE; Singh, WE].

In the closed Friedmann universe, the 'K' loop quantization provides a better approximation to the 'field strength' loop quantization [Corichi, Karami; Singh, WE], so here I will consider the 'K' loop quantization.

Spatial Curvature in LQC

In the presence of spatial curvature, the holonomy of the field strength around a loop of minimal area is not almost-periodic in the connection, so a different approach is required.

So far, there are two suggestions: 'A' and 'K' loop quantizations based on the parallel transport of the Ashtekar-Barbero connection and the extrinsic curvature respectively [Vandersloot; Ashtekar, WE; Singh, WE].

In the closed Friedmann universe, the 'K' loop quantization provides a better approximation to the 'field strength' loop quantization [Corichi, Karami; Singh, WE], so here I will consider the 'K' loop quantization.

The Bianchi IX effective constraint for the 'K' loop quantization is

$$\mathcal{C}_H \sim \mathcal{C}_H^{(LQC\ \ B.I)} + e^{4\alpha_1} + e^{4\alpha_2} + e^{4\alpha_3}.$$

LQC affects only the Bianchi I part, the potential is unchanged.

Approximations

Recall from the discussion above that the Kasner transitions generated by the spatial curvature occur rapidly and it is reasonable to approximate these transitions as occuring instantaneously; away from the transitions the spatial curvature is negligible.

The LQC bounce happens even more rapidly.

Therefore, unless initial conditions are carefully chosen so that the spatial curvature is important during the LQC bounce, for typical solutions:

The spatial curvature is negligible during the LQC bounce, and LQC effects are negligible during the Mixmaster-type Kasner transitions.

Approximations

Recall from the discussion above that the Kasner transitions generated by the spatial curvature occur rapidly and it is reasonable to approximate these transitions as occuring instantaneously; away from the transitions the spatial curvature is negligible.

The LQC bounce happens even more rapidly.

Therefore, unless initial conditions are carefully chosen so that the spatial curvature is important during the LQC bounce, for typical solutions:

The spatial curvature is negligible during the LQC bounce, and LQC effects are negligible during the Mixmaster-type Kasner transitions.

For these solutions, the transition rule at the LQC bounce will be the same as for Bianchi I, and the Mixmaster transition rules will be the same as in general relativity.

LQC Transition Rules for the Kasner Parameters

From the transition map $k_i \to \tilde{k}_i = \frac{2}{3} - k_i$, it immediately follows that

$$u \to \tilde{u} = \frac{u+2}{u-1}, \qquad p_\Omega \to \tilde{p}_\Omega = -p_\Omega .$$

LQC Transition Rules for the Kasner Parameters

From the transition map $k_i \to \tilde{k}_i = \frac{2}{3} - k_i$, it immediately follows that

$$u \to \tilde{u} = \frac{u+2}{u-1}, \qquad p_\Omega \to \tilde{p}_\Omega = -p_\Omega.$$

A little work is required to derive the transition maps for v and κ. Under the assumptions that (i) the last Mixmaster Kasner transition before the bounce occured when one $\alpha_i = 0$, and (ii) the LQC bounce occurs instantaneously when $\theta = \theta_{\rm crit} \sim \ell_{\rm Pl}^{-1}$,

LQC Transition Rules for the Kasner Parameters

From the transition map $k_i \to \tilde{k}_i = \frac{2}{3} - k_i$, it immediately follows that

$$u \to \tilde{u} = \frac{u+2}{u-1}, \qquad p_\Omega \to \tilde{p}_\Omega = -p_\Omega.$$

A little work is required to derive the transition maps for v and κ. Under the assumptions that (i) the last Mixmaster Kasner transition before the bounce occured when one $\alpha_i = 0$, and (ii) the LQC bounce occurs instantaneously when $\theta = \theta_{\rm crit} \sim \ell_{\rm Pl}^{-1}$,

$$v \to \tilde{v} = \frac{2(2u+1)\,\ln\left(2\,\gamma\sqrt{\Delta}\,|p_\Omega|\right) + 3\kappa}{2(u+1)\,\ln\left(2\,\gamma\sqrt{\Delta}\,|p_\Omega|\right) + v\kappa},$$

$$\kappa \to \tilde{\kappa} = -\frac{2(u+1)}{u-1}\ln\left(2\,\gamma\sqrt{\Delta}\,|p_\Omega|\right) - \frac{\kappa v}{u-1}.$$

Some Comments

- The derivation of the transition rules for u and p_Ω required less assumptions and so are more robust.

- The Planck scale only appears in the transition rules for v and κ; the transition rules for u and p_Ω are independent of $\ell_{\rm Pl}$. The only input needed for deriving the transition rules for u and p_Ω is that the equations of motion for the Π_i are all identical.

- The transition rules are the same no matter the initial values of (u, v, p_Ω, κ).

Chaos?

The classical Bianchi IX dynamics are known to be chaotic [Barrow, Chernoff; Cornish, Levin]. What about the LQC effective dynamics?

Classically, the chaos arises due to the repeated Kasner era changes when $u \to \tilde{u} = 1/(u-1)$, with $u - 1 < 1$; there are an infinite number of these for Bianchi IX in general relativity. An important difference in LQC is that there will be a finite number of such era changes in any bounce/recollapse cycle.

Chaos?

The classical Bianchi IX dynamics are known to be chaotic [Barrow, Chernoff; Cornish, Levin]. What about the LQC effective dynamics?

Classically, the chaos arises due to the repeated Kasner era changes when $u \to \tilde{u} = 1/(u-1)$, with $u - 1 < 1$; there are an infinite number of these for Bianchi IX in general relativity. An important difference in LQC is that there will be a finite number of such era changes in any bounce/recollapse cycle.

Note that the usual BKL transition map is unmodified by LQC effects; rather LQC adds the bounce, a new type of Kasner transition.

And there are an infinite number of bounce/recollapse cycles, so it appears likely that the Bianchi IX space-time will be chaotic in LQC also. However, this is a statement about the full sequence of bounce/recollapse cycles, not about one such cycle alone.

The BKL Conjecture

The BKL conjecture is that near a space-like singularity time-like derivatives dominate over space-like derivatives, and the dynamics at a generic space-time point will be those of a Bianchi IX model.

A number of caveats are important:

- Not all singularities are space-like, in particular weak null singularities appear to be generic in general relativity,
- The BKL behaviour arises asymptotically, potentially past the Planck scale in which case it would be irrelevant for LQC,
- 'Spike surfaces' are known to arise where spatial derivatives remain important,
- The effective equations may break down when considering the dynamics at a point as quantum fluctuations may play an important role.

Potential Implications for Singularities

With these caveats in mind, what insight can we gain?

In regions where the BKL behaviour arises and quantum gravity effects are captured by the LQC effective dynamics:

- Singularities will be resolved point by point, with the expansion and shear bounded by the Planck scale,
- The dynamics will be sensitive to initial conditions at each Kasner era change.

Potential Implications for Singularities

With these caveats in mind, what insight can we gain?

In regions where the BKL behaviour arises and quantum gravity effects are captured by the LQC effective dynamics:

- Singularities will be resolved point by point, with the expansion and shear bounded by the Planck scale,
- The dynamics will be sensitive to initial conditions at each Kasner era change.

Will this generate uncontrollably large inhomogeneities?

Potential Implications for Singularities

With these caveats in mind, what insight can we gain?

In regions where the BKL behaviour arises and quantum gravity effects are captured by the LQC effective dynamics:

- Singularities will be resolved point by point, with the expansion and shear bounded by the Planck scale,
- The dynamics will be sensitive to initial conditions at each Kasner era change.

Will this generate uncontrollably large inhomogeneities?

This will depend on the number of Kasner era changes. Previous studies indicate a small number between the onset of BKL behaviour and the Planck scale [Doroshkevich, Novikov; Weaver, Isenberg, Berger; Garfinkle].

This suggests that inhomogeneities will grow, but not uncontrollably.

Discussion

- The bounce in the effective LQC dynamics can be viewed as a Kasner transition with the simple rule $k_i \to \tilde{k}_i = \frac{2}{3} - k_i$ relating the Kasner exponents of the pre- and post-bounce branches.
- Transition rules for the LQC bounce can be derived for (u, v, p_Ω, κ) that parameterize each Kasner epoch in the Mixmaster dynamics; this provides a quantum gravity extension to the Mixmaster map.
- The Bianchi IX space-time is likely to be chaotic for the sequence of all bounce/recollapse cycles.
- The BKL dynamics, where applicable, will likely not generate uncontrollably large inhomogeneities due to the limited number of Kasner transitions near the LQC bounce.

Outlook

There remain many open problems here, two that I think are particularly important are:

- Study numerical solutions of the LQC effective dynamics for the Bianchi II and Bianchi IX space-time to check the assumptions used in the derivations.
- Understand how to do a loop quantization of non-perturbative inhomogeneities (which is tractable for calculations/numerics), perhaps building off the BKL conjecture. The reformulation of the BKL conjecture in LQG-like variables [Ashtekar, Henderson, Sloan] could be a good place to start.

Outlook

There remain many open problems here, two that I think are particularly important are:

- Study numerical solutions of the LQC effective dynamics for the Bianchi II and Bianchi IX space-time to check the assumptions used in the derivations.
- Understand how to do a loop quantization of non-perturbative inhomogeneities (which is tractable for calculations/numerics), perhaps building off the BKL conjecture. The reformulation of the BKL conjecture in LQG-like variables [Ashtekar, Henderson, Sloan] could be a good place to start.

Thank you for your attention!