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Petrovsky condition for forward evolution and semigroups

About ten years ago suddenly I get infected by interest in theory of distributions, especially in rapidly decreasing distributions on $\mathbb{R}^n$ whose set is denoted by $O'_C(\mathbb{R}^n)$. During these ten years I proved the following:

$$O'_C(\mathbb{R}^n) * S(\mathbb{R}^n) \subset L(S(\mathbb{R}^n), S(\mathbb{R}^n)),$$

so that $O'_C(\mathbb{R}^n)$ can be equipped with topology op induced from $L(S(\mathbb{R}^n), S(\mathbb{R}^n))_b$. The locally convex space $(O'_C(\mathbb{R}^n), op)$ is complete. The Fourier transformation yields an isomorphism of locally convex spaces $(O'_C(\mathbb{R}^n), op)$ and $O_M(\mathbb{R}^n)$ such that $F(O'_C(\mathbb{R}^n) * S'(\mathbb{R}^n)) = F(O'_C(\mathbb{R}^n)) \bullet F(S'(\mathbb{R}^n))$. Thanks to the fact that $O_M(\mathbb{R}^n)$ is algebra of multipliers of $S(\mathbb{R}^n)$ the last isomorphism implies at once that for $(O'_C(\mathbb{R}^n), op)$ the fundamental Theorem XV from Chapter VII of the book of L.Schwartz is true. The topology β in $O'_C(\mathbb{R}^n)$, invented originally by L.Schwartz, is strictly finer than the topology op, so that $(O'_C(\mathbb{R}^n), \beta)$ is not isomorphic with $O_M(\mathbb{R}^n)$. The aforementioned relation between the Petrovsky condition for forward evolution and one-parameter convolution semigroups (in $O'_C(\mathbb{R}^n)$ and in $S'(\mathbb{R}^n)$) has secondary importance, but agrees with scope of the conference.