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Convergence of Positive Semigroups and Hyper-Bounded Operators

If $(T_t)_{t\in(0,\infty)}$ is a positive and bounded operator semigroup on L^p and if the essential spectral radius $r_e(T_t)$ of at least one operator T_t is strictly smaller than 1, a classical theorem of Lotz ensures operator norm convergence of T_t as time tends to infinity. This is one reason (among many others) why one is interested in criteria which ensure the property $r_e(T) < 1$ for a positive operator T.

Let (Ω, μ) be a finite measure space and let $p \in (1, \infty)$. A bounded linear operator T on $L^p := L^p(\Omega, \mu)$ is called *hyper-bounded* if $TL^p \subseteq L^q$ for some $q > p$. In 2015 L. Miclo [1] showed that a positive and hyper-bounded operator T on L^2 fulfils $r_e(T) < 1$ in case that T has spectral radius 1 and is self-adjoint (and fulfils a few technical assumptions); this solved a long open conjecture of Høegh-Krohn and Simon.

In this talk we demonstrate that the same theorem remains true under more general assumptions: a positive and hyper-bounded operator T on L^p which is merely power-bounded always fulfils $r_e(T) < 1$. Our methods are very different from Miclo's; we rely on an ultra power technique, combined with the fact that L^p and L^q are not isomorphic for $p \neq q$.

References

[1] Laurent Miclo, *On hyperboundedness and spectrum of Markov operators*, Invent. Math. **200** (2015), no. 1, 311–343.