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Existence of invariant densities for conservative linear kinetic equations on the torus without spectral gaps

This work is the continuation of a general theory, given in *J. Funct. Anal,* **266 (11)** (2014), on time asymptotics of conservative linear kinetic equations on the torus exhibiting a spectral gap. We consider neutron transport-like equations on $L^1(\mathcal{T}^n \times V)$ $(n \geqslant 1)$ where $\mathcal{T}^n := \mathbb{R}^n/(\mathbb{Z})^n$ is the n-dimensional torus under the *conservativity* assumption $\sigma(x,v) = \int_V k(x,v',v)\mu(dv')$ where σ is the collision frequency and k is the scattering kernel while μ is a velocity Radon measure on $\mathbb{R}^n$ with support V. The "collisionless" equation on $\mathcal{T}^n \times V$ is governed by a weighted shift C_0-semigroup $(U(t))_{t\geq 0}$ whose type (or growth bound) $\omega(U) < 0$ *if and only if* there exist $C_1 > 0$ and $C_2 > 0$ such that

$$\int_0^{C_1} \sigma(x+sv,v)ds \geqslant C_2 \text{ a.e. on } \mathcal{T}^n \times V. \quad (1)$$

The full dynamics is governed by a *stochastic* (i.e. mass-preserving on the positive cone) C_0-semigroup $(W(t))_{t\geq 0}$. Under very general assumptions, $(U(t))_{t\geq 0}$ and $(W(t))_{t\geq 0}$ have the same *essential* type $\omega_{ess}(W) = \omega_{ess}(U)$. In particular, under (1) $\omega_{ess}(W) < 0 = \omega(W)$ i.e. $(W(t))_{t\geq 0}$ exhibits a *spectral gap* and 0 is an isolated eigenvalue of $T+K$ with finite algebraic multiplicity. In particular $(W(t))_{t\geq 0}$ tends exponentially to the spectral projection associated to the 0 eigenvalue of the generator. The object of the present work is to consider the critical case $\omega(U) = 0$, i.e. when $\sigma(.,.)$ vanishes on some characteristic curve. In this case, $(W(t))_{t\geq 0}$ has *not* a spectral gap. We provide general tools to study the existence of an invariant density.