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Approximation of fractional differential equations in Banach spaces

In this talk, we present our research on convergence of difference schemes for fractional differential equations. Using implicit difference scheme and explicit difference scheme, we have a deal with the full discretization (in space and in time) of the solutions of fractional differential equations in Banach spaces. A lot of papers were devoted to the discretization of C_0-semigroups in traditional way. Recently, we considered the discrete approximation of integrated semigroups [1], where the order of convergence was obtained using ill-posed problems theory. In this talk we continue our investigations [2] on discretization of differential equations of fractional order $0 < \alpha < 1$ in Banach spaces and get the optimal [3] order of convergence $O(\tau_n^\alpha)$.

We develop such approach and consider approximation of semilinear equations in the form

$$D^\alpha u(t) = Au(t) + f(u(t)),\, u(0) = u^0,$$

where $D^\alpha u(t)$ is a fractional derivative of $u(t)$ in Caputo sense and function $f(\cdot)$ is smooth enough. The approximation of such problems is based on the principle of compact approximation.

References

[1] V. Morozov M. Li and S. Piskarev, *On the approximations of derivatives of integrated semigroups*, J. Inverse Ill-Posed Probl. **18** (2010), no. 5, 515–550.

[2] M. Li R. Liu and S. Piskarev, *Stability of difference schemes for fractional equations*, Differ. Equ. **51** (2015), no. 7, 904–924.

[3] ______, *The order of convergence of difference schemes for fractional equations*, Numerical Functional Analysis and Optimization **38** (2017), no. 6, 754–769.