

Regularity results for fractional Patlak-Keller-Segel system

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Abstract

We consider the parabolic-elliptic Patlak-Keller-Segel system with fractional diffusion

$$\begin{cases} u_t + (-\Delta)^{\frac{\alpha}{2}} u &= -\chi \operatorname{div}(u \nabla v) + f(u) \\ -\Delta v &= u - \langle u_0 \rangle. \end{cases}$$

and its certain generalisations. The following results, obtained jointly with Rafael Granero-Belinchón, will be presented

- Disproof of the finite-time blowup conjecture in the critical, 1d case, see [1], [2].
- Global-in-time smoothness results in 1d and 2d for the logistically damped case ($f(s) = rs(1-s)$), but with both diffusions and damping weak [3, 4].
- Criteria for homogenous asymptotics [5].
- Well-posedness in critical spaces (work in progress).

References

- [1] J. Burczak and R. Granero-Belinchón. Keller-Segel meets Burgers on S^1 : large-time smooth solutions. *Nonlinearity*, 29:3810–3836, 2016.
- [2] N. Bournaveas and V. Calvez. The one-dimensional Keller-Segel model with fractional diffusion of cells. *Nonlinearity*, 23, 2010.
- [3] J. Burczak and R. Granero-Belinchón. Global solutions for a supercritical drift-diffusion equation. *Adv. Math.*, 295:334–367, 2016.
- [4] J. Burczak and R. Granero-Belinchón. Suppression of blow up by a logistic source in 2D Keller-Segel system with fractional dissipation. *J. Differential Equations*, 263:6115–6142, 2017.
- [5] J. Burczak and R. Granero-Belinchón. Boundedness and homogenous asymptotics for a fractional, logistic Keller-Segel equations. *to appear in DCDS-S*.