No critical mass phenomenon in 1d quasilinear fully parabolic Keller-Segel system

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Abstract

It is known since 2010 (see Cieslak and Laurencot, DCDS 2010) that in a parabolic-elliptic quasilinear Keller-Segel system critical nonlinear diffusion does not exist. Either all the solutions are global and bounded or one finds solutions blowing up in finite time regardless the magnitude of initial mass. I will show recent results concerning the similar phenomenon in the fully parabolic case.

Namely, I will prove that also in the fully parabolic case the 'critical' diffusion $1/(1+u)$ leads to global bounded solutions no matter what initial data do we start with. Our proof relies on a new Lyapunov-like functional we found in [1]. It gives enough information to arrive at global existence, [1] . Moreover, several new space-time estimates regarding the fully parabolic Keller-Segel (including a second new Lyapunov-like functional) led us to the boundedness of solutions, see [2].

References

[1] T.Cieślak, K. Fujie, No critical nonlinear diffusion in 1D quasilinear fully parabolic chemotaxis system, to appear in Proc. Amer. Math. Soc.

[2] B. Bieganowski, T.Cieślak, K. Fujie, Boundedness of solutions to the critical fully parabolic quasilinear one-dimensional Keller-Segel system, arXiV:1712.07876