Infinite time blow-up of many solutions
to a general quasilinear parabolic-elliptic Keller-Segel system

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Abstract

We consider a parabolic-elliptic chemotaxis system generalizing

$$\begin{cases} u_t = \nabla \cdot ((u+1)^{m-1}\nabla u) - \nabla \cdot (u(u+1)^{\sigma-1}\nabla v) \\ 0 = \Delta v - v + u \end{cases}$$

in bounded smooth domains $\Omega \subset \mathbb{R}^N$, $N \geq 3$, and with homogeneous Neumann boundary conditions. We show that

*) solutions are global and bounded if $\sigma < m - \frac{N-2}{N}$

*) solutions are global if $\sigma \leq 0$

*) close to given radially symmetric functions there are many initial data producing unbounded solutions if $\sigma > m - \frac{N-2}{N}$.

In particular, if $\sigma \leq 0$ and $\sigma > m - \frac{N-2}{N}$, there are many initial data evolving into solutions that blow up after infinite time.